

Summer Semester 2014

## Assignment on Massively Parallel Algorithms - Sheet 6

Due Date 11. 06. 2014

### Exercise 1 (Matrix Vector Multiplication, 5 Credits)

In the given Framework `MatrixVectorMul` Matrix  $A$  is stored using a row major order.

Your tasks are the following:

- Implement a Matrix Vector multiplication kernel for the above Matrix stored in row major order.
- Implement a method to store the above Matrix in column major order and then modify the above Matrix vector multiplication kernel to handle matrix stored in column major order .
- Compare run times between the above two implementations (**row major order vs column major order**) for different Matrix sizes and provide arguments for the differences/similarities between run times for these two implementations .

### Exercise 2 (Matrix Multiplication for APSP, 5 Credits)

Given a directed graph  $G = (V, E)$  with distance function  $dist : E \rightarrow \mathbf{R}$ ,  $|V| = n$  where  $V$  is the vertices (or nodes) set and  $E$  edges set. The adjacency Matrix of the above given directed graph is defined as follows:

$n \times n$  matrix  $A = (\delta_{ij})$  of edge distances :

$$\delta_{ij} = \begin{cases} dist(v_i, v_j), & \text{if } (v_i, v_j) \in E \\ \infty, & \text{if } (v_i, v_j) \notin E \\ 0, & \text{if } i = j \end{cases} \quad (1)$$

Given  $D^1 = A$ , compute a series of matrices  $D^2, D^3, \dots, D^{n-1}$  ( generated by algorithm *EXTEND-PATH*( $D^{m-1}, A$ ) see Algorithm 2 ), where  $D^k = (d_{ij}^k)$  for  $m = 1, 2, \dots, n - 1$  contains the shortest path distances between each pair of vertices  $v_i, v_j$  with at most  $k$  edges Then the final matrix  $D^{n-1}$  contains actual shortest path distances. Simple algorithm for computing  $D^{n-1}$  see Algorithm 1

The given APSP framework computes the matrix product of two matrices  $A$  ( $8 \times 8$ ) and  $B$  ( $8 \times 8$ ) and then resulting matrix  $C = c_{ij}$  is displayed using color coding as follows.

$$color_{ij} = \begin{cases} (r = 0, g = 0, b = 0) & \text{if } c_{ij} = 0 \\ (r = 2 * c_{ij}/20, g = 0, b = 0), & \text{if } 0 < c_{ij} \leq 10 \\ (r = 1.0, g = c_{ij}/20, b = 0), & \text{if } 10 < c_{ij} \leq 20 \\ (r = 0, g = 1.0, b = 0), & \text{if } c_{ij} > 20 \end{cases} \quad (2)$$

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**Algorithm 1** Slow APSP Algorithm

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```
1: procedure SLOW-APSP( $A$ )
2:    $D^1 \leftarrow A$ 
3:   for  $k \leftarrow 0$  to  $n - 1$  do
4:      $D^k \leftarrow \text{EXTEND-PATH}(D^{k-1}, A)$ 
5:   end for
6:   return  $D^{n-1}$ 
7: end procedure
```

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**Algorithm 2** Method for computing the next D matrix

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```
1: procedure EXTEND-PATH( $D, A$ )
2:    $D = d_{ij}$  is an  $n \times n$  matrix
3:   for  $i \leftarrow 1$  to  $n$  do
4:     for  $j \leftarrow 1$  to  $n$  do
5:        $d_{ij} \leftarrow \infty$ 
6:       for  $l \leftarrow 1$  to  $n$  do
7:          $d_{ij} \leftarrow \min(d_{ij}, d_{il} + \delta_{lj})$ 
8:       end for
9:     end for
10:  end for
11:  return  $D$ 
12: end procedure
```

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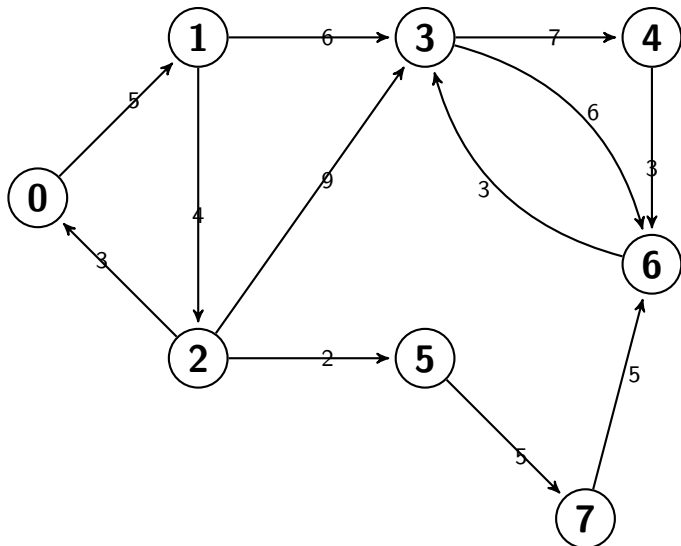


Figure 1: Graph

Your task is to modify the APSP framework in order to compute the shortest path matrix for the graph given above see Figure 1. You need `freelut` for the framework. For installation please see the slides from the first tutorial.

*Hint:* Please note that the tiled version of Matrix Multiplication is used in the above given framework and use the similarities between algorithm EXTEND-PATH and Matrix multiplication algorithm to modify the above framework for computing All pairs shortest path as discussed in the lecture..