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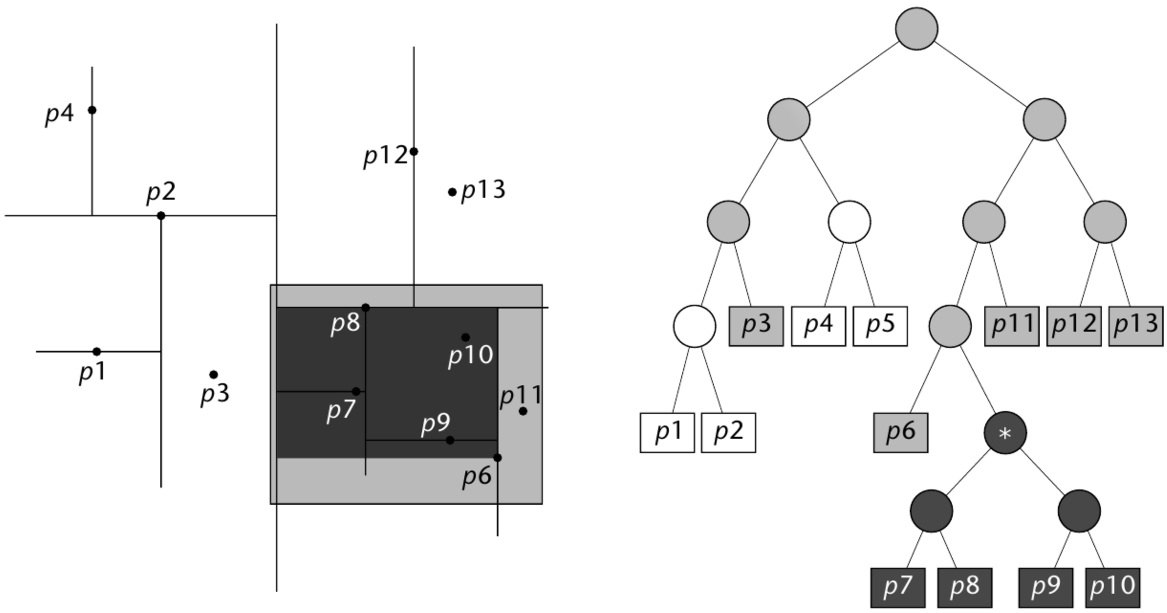
Assignment on Computational Geometry - Sheet 3

Due Date 04. 06. 2021

Due by 04. 06. 2021 via email to weller@informatik.uni-bremen.de)

Exercise 1 (Range Queries, *3+3 Credits*)

kd-trees are perfectly suited for range queries .



a) Describe an algorithm that delivers all points in a pre-defined axis-aligned query rectangle.

b) What is the (output-sensitive) running time of this algorithm?

Exercise 2 (kd-Trees and Triangles, *3+4 Credits*)

It is also possible to use kd-trees for triangular range queries.

a) Show that the running time for a triangular range query in a 2-dimensional kd-tree can be linear in the worst case, even if the triangle does not contain a single point of the point set.

   Hint: Look at points on the line $y = x$.

b) In the following we will only consider triangles in the plane with edges that are either axis-aligned (horizontally or vertically) or that have a gradient of -1 or $+1$.

   Define a data structure that can answer range-queries for this particular triangles more efficiently.

   Hint: Extend the 2-dimensional kd-tree.

Exercise 3 (Beyond General Position, *2+3+2 Credits*)

Until now, we mainly considered a *good* distribution of the points: they are in the so called *general position*, i.e. all points differ in their x- and y-coordinates. Consequently, it is always possible to split them with an axis-aligned splitting line into two subsets of (almost) the same size. Unfortunately, in real-world data sets, points hardly have this nice property. Therefore, we also have to think about the implications that such *bad* data sets have on the performance of our data structures.

a) Describe an example for a set of points in 2D that you cannot divide into two sub-sets of the same size with *any* axis-aligned line (Obviously, your set should have even size).

b) Let R be a set of points in the 2D plane. Show that there always exist an axis-aligned line that does not contain any point of R but splits R into two subsets where the smaller subset contains at least $\frac{n-1}{4}$ points. Is this bound tight?

c) Has the bound for the split in part (b) any negative implications on the theoretical running time of range queries in kd-trees?