

Summer Term 2023

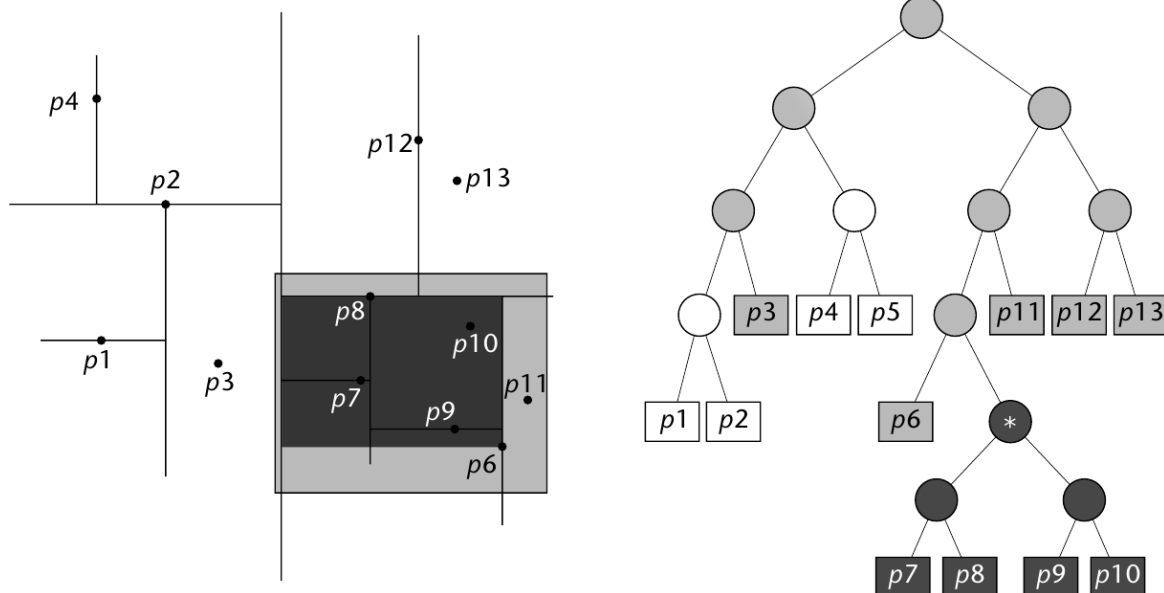
Assignment on Computational Geometry - Sheet 2

Due Date 15. 05. 2023

Due by 15. 05. 2023 via email to weller@informatik.uni-bremen.de

Exercise 1 (Range Queries, 3+3 Credits)

kd-trees are perfectly suited for range queries .



- Describe an algorithm that delivers all points in a pre-defined axis-aligned query rectangle.
- What is the (output-sensitive) running time of this algorithm?

Exercise 2 (*kd*-Trees and Triangles, 3+4 Credits)

It is also possible to use *kd*-trees for triangular range queries.

- Show that the running time for a triangular range query in a 2-dimensional *kd*-tree can be linear in the worst case, even if the triangle does not contain a single point of the point set.
Hint: Look at points on the line $y = x$.
- In the following we will only consider triangles in the plane with edges that are either axis-aligned (horizontally or vertically) or that have a gradient of -1 or $+1$.
Define a data structure that can answer range-queries for this particular triangles more efficiently.
Hint: Extend the 2-dimensional *kd*-tree.

Exercise 3 (Beyond General Position, 2+3+2 Credits)

Until now, we mainly considered a *good* distribution of the points: they are in the so called *general position*, i.e. all points differ in their x - and y -coordinates. Consequently, it is always possible to split them with an axis-aligned splitting line into two subsets of (almost) the same size. Unfortunately, in real-world data sets, points hardly have this nice property. Therefore, we also have to think about the implications that such *bad* data sets have on the performance of our data structures.

- a) Describe an example for a set of points in 2D that you cannot divide into two sub-sets of the same size with *any* axis-aligned line (Obviously, your set should have even size).
- b) Let R be a set of points in the 2D plane. Show that there always exist an axis-aligned line that does not contain any point of R but splits R into two subsets where the smaller subset contains at least $\frac{n-1}{4}$ points. Is this bound tight?
- c) Has the bound for the split in part (b) any negative implications on the theoretical running time of range queries in kd -trees?