True Grid

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Abstract. The Renaissance architect, moral philosopher, cryptographer, mathematician, Papal adviser, painter, city planner and land surveyor Leon Battista Alberti provided the theoretical foundations of modern perspective geometry. Alberti's work on perspective exerted a powerful influence on painters of the stature of Albrecht Dürer, Leonardo da Vinci and Piero della Francesca. But his *Della pittura* of 1435–36 contains also a hitherto unrecognized ontology of pictorial projection. We sketch this ontology, and show how it can be generalized to apply to representative devices in general, including maps and spatial and non-spatial databases.

Fig. 1 Albrecht Dürer's interpretation of 'The Draftsman's Net'

1 Through a Glass Clearly

The *Della pittura* of the Renaissance artist and art theorist Leon Battista Alberti, dating from 1435–36, is the first modern treatise on painting. It defends a view according to which the proper goal of the artist is to produce a picture that will represent the visible world *as if the observer of the picture were looking through a window*.

This open window conception reflects a time when painting is still an adjunct of architecture: paintings are designed to enhance one's home. The aesthetic experience of a building's interior and the aesthetic experience of the paintings on its walls are meant to be fused into one:

> the picture must be so painted that a spectator's imagination is drawn towards the wall-plane, not away from it. This is why Renaissance painters, acting as interior decorators, revived and elaborated the system of perspective already used by interior decorators at Pompeii and elsewhere in the ancient world. (Collingwood 1938, p. 153)

Alberti's conception of the painting was extremely influential. Indeed the art historian Erwin Panofsky argues that, while there are elements of perspectival foreshortening in earlier works of art, one can properly speak of a perspectival intuition of space only where a 'whole picture is as it were transformed into a "window" through which we should then believe ourselves to be looking into the space'. (Panofsky 1927) To rub the same point home, Dürer, in his woodcuts, always depicts the process of perspective projection in such a way that this process is situated in a room in which a section of window clearly appears. ('Perspective' means, roughly, 'seeing through' or 'seeing clearly'.)

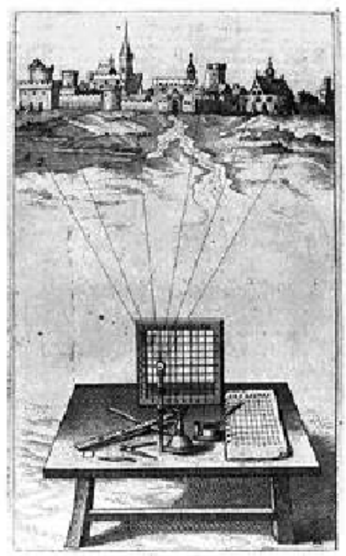

Fig. 2 Alberti's *Reticolato*

Alberti presented his ideas on perspective in terms of his so-called '*reticolato*', also known as Alberti's 'grid' or 'grill' (*graticola*), a mechanical aid to painters in the execution of the *fenestra aperta* technique, which involved creating a grid across an actual window in order to enable the artist to transfer the scene visible through the window to a correspondingly gridded canvas. Because parallax is here so strong, it is unlikely that such devices were ever in fact used by painters. Even the slightest movement on the painter's part will have a dramatic effect upon the scene perceived. We might think of the *reticolato*, rather, as a pedagogical device, designed to help the artist understand how perspective works.

Figure 2 depicts rays extending from the abstractly represented (single) eye of the artist, passing out through the cells of the artist's grid and forming a visual pyramid along their way to their final destination: an array of planes in the background of the figure. To the right of the grid is a correspondingly gridded notepad to which the artist is supposed to transfer the contents of each successive cell, contents that have been 'measured' by the rays, which reach out like feelers to touch the corresponding portions of reality. In this way the artist can apprehend in systematic and accurate fashion the visual qualities in the scene before him. Dürer's treatise on measurement, his *Underweysung der Messung* of 1527, illustrates a range of similar machines by which an artist might 'scientifically' depict people and objects along these same lines. The machines employ a glass plate or frame divided into small squares by a net or veil of black thread. This allows the imagined artist to locate marks within the space of the painting in such a way that their shapes, sizes and relative positions conform to what we would see if we were observing corresponding objects in reality. At the same time the grids encourage a new way of seeing, through which a portion of the visible world is organized into a geometric composition.

2 Theatrum Orbis Terrarum

The *practical* problem of projecting an array of objects existing in three-dimensional space onto a two-dimensional plane was solved at around the time of Brunelleschi, who is held to have created the first painting – of the Baptistery of St. John in Florence – in 'true perspective', sometime between 1415 and 1425. The problem was solved *theoretically* by Alberti in Book I of *Della pittura*, which presents the mathematical theory of the way in which a plane intersects a visual pyramid in exactly the way that is captured intuitively in images of the *reticolato*. Over the next century and a half Brunelleschi's and Alberti's work, and that of their contemporaries and successors, including not only Dürer but also Piero della Francesca and Leonardo da Vinci (all of whom were influenced by Alberti), transformed painting in a way which enabled European art for the first time to free itself from the inhibiting burden of those earlier traditions of visual representation which had remained unaware of perspective.

The theoretical solution of the problem of perspective put forward in *Della pittura* was a scientific discovery of the first importance, and it ranks with the later contribution of Desargues in launching our contemporary understanding of projective geometry. But why did mankind have to wait until the fifteenth century, 1700 years after Euclid's *Elements* and *Optics*, to take what Panofsky calls 'the apparently small step of intersecting the visual pyramid with a plane'? How can this be lag explained, if perspective had been present in all seeing from the very start? Samuel Edgerton, in his *The Renaissance Rebirth of Linear Perspective*, presents a two-part solution to this problem, holding 1. that there arose among a certain group of citizens of Florence in the early years of the fifteenth century a new way of apprehending visual space as a structure ordered by an abstract uniform system of linear coordinates, and 2. that the decisive impetus towards this new was of seeing was inspired by developments in cartography, and specifically by the rediscovery of Ptolemy's *Geographia*, a work dating from around 140 A.D., which arrived in Florence in 1400 to great acclaim.

In more traditional metaphysical systems, such as were employed, for example, by Aristotle, a distinction had been drawn between the realm of astronomy, which is subject to precise, intelligible mathematical laws, and the sublunar world of change and decay, which is only partially intelligible to mortals such as ourselves. The principal achievement of Ptolemy's *Geographia* turned on its demonstration of the possibility of using a regular mathematical grid system to map the entire known world. Ptolemy thereby showed how the earth below could be comprehended in a uniform way in terms of a single mathematical system. Essential to this achievement was the idea that the grid not only have the mathematical properties of an exhaustive tessellation, but also that it be *transparent*. Ptolemy's grid is not a part of any of reconstruction of some abstract mathematical realm. It is designed, rather, to help us to grasp this world, the world of sensate matter, as it really is.

The impact of Ptolemy's transparent grid system was so great that already by 1424 Florence has acquired the reputation of a center of cartographic and geographic study, and its influence may have extended, through commentaries on Florentine versions of the *Geographia*, to Christopher Columbus. Ptolemy's grid system also began to be taken up as a basis for territorial boundary-demarcation. Certainly grid systems had been used for surveying purposes since much earlier times, above all by the *agrimensores* who had introduced centuriation into many parts of the Roman Empire. But like the grids used in the seventeenth century in dividing up the Dutch polders Roman centuriation applied always to the demarcation of intraterritorial lines. During the wars of 1420, however, a longitudinal line was proposed as the boundary between the two states of Milan and Florence. Edgerton (1975, p. 115) conjectures that this may have been the first occasion when an imaginary mathematical line – a fiat boundary – was recognized as a political-territorial limit.

As Veltman (1977) points out, there are a number of problems with the details of Edgerton's account. Yet the similarities between Ptolemy's method of projecting arcs of circles visible on a globe onto a planar map and the method of perspective painting encapsulated in Alberti's *reticolato* are strikingly close, and the hypothesis that Alberti recognized the significance of Ptolemy's cartographic projection method for painting is supported further by Alberti's own claims on behalf of his *reticolato*, for example that it 'affords the greatest assistance in executing your pictures, since you can see any object that is round and in relief, represented on the flat surface of the veil.' (Alberti 1435/36, pp. 68 f.)

In his introduction to the English translation of *Della pittura*, Spencer conjectures (1956) that Alberti may have arrived at his solution to the problem of perspective also through his own experiences in the domain of surveying.[1] Between 1431 and 1434, which is to say just before the completion of *Della pittura*, Alberti composed a small work entitled *Descriptio urbis Romae* in which he sets forth both a method for surveying and a table of sightings obtained in applying this method to yield what he calls a 'picture' of Rome.[2] A surveyor needs some means to determine the proportionate distance between any two quantities. There can be no doubt that Alberti understood such a method,[3] and one which did not make use of trigonometry, which had not yet been invented. Further evidence that surveying is a source of Alberti's construction is provided by the privileged role awarded by Alberti to the measure of a staff held at arm's length. Spencer points out further that Piero della Francesca gives an account of a perspective construction – based on

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[1] The association between optics and surveying has a long tradition, as is shown not least by the inclusion of four theorems on surveying in Euclid's *Optics.* The philosopher Al-Farabi could write of *optics* in the tenth century that it 'makes it possible for one to know the measurement of that which is far distant, for example, the height of tall trees and walls, the width of valleys and rivers, the height of mountains and the depth of valleys, rivers' (from Veltman 1999).

[2] An account of the instrument which Alberti invented for this purpose is given by Spencer as follows:

> a bronze disc [is] mounted parallel to the surface of the earth and divided on the circumference into 48 degrees. At the centre a metal or wooden ruler, divided into 50 degrees, is pivoted. … When the ruler is placed at right angles to the line of sight, it becomes possible to compute the distance of the object – given its width – or its actual width – given the distance – by means of the similarity of triangles. (Spencer 1956, pp. 113 f.)

3 In his *Ludi mathematici* composed for Borso d'Este about 1450 he demonstrates the well known operation of determining the width of a stream by means of a staff and the similarity of triangles. (Spencer, 1956, pp. 114 f.)

plan and elevation drawings connected by lines from a point of sight and cut by a perpendicular – which is essentially Alberti's surveying method from the *Descriptio urbis Romae* moved indoors to the drawing board.

3 Fiat Lux

Alberti's contribution to the history of cartography has been noted by others. Our purpose here is to show that *Della pittura* contains also a contribution to our understanding of the ontology of pictures which can be generalized to projective devices in general. There are, according to Alberti, two kinds of matter with which the painter must be concerned. On the one hand is the three-dimensional matter of the observable world, which exists in space and light. On the other hand is the two-dimensional matter of the painting, a simulacrum of reality that is produced by the painter, who 'must find a means of controlling the matter of the macrocosm if he is to represent it in his microcosm.' (Spencer 1956, p. 19) The first kind of matter is composed of surfaces in three-dimensional reality, the second of marks the artist makes on the flat plane of the canvas. (Compare Gibson 1980) This second kind of matter exists, if the artist is successful, in the form of a visual story (*istoria*) that is constructed out of points, lines and planes (marks) on a panel or canvas. The latter are grouped together to form (for example) limbs, bodies and groups of bodies related together in a way that is analogous, as Alberti sees it, to the way in which words, phrases, sentences and paragraphs are related together in natural language. Alberti develops rules for manipulating these various elements in an *istoria*, based on the four principles of *dignità*, *varietà*, *modestia* and *verisimilitudo*. Together with geometry, these principles constitute the basis of a rational art or indeed of a science of painting.

No painter can paint well who does not know geometry.[4] The observed scene, the scene that is visible and that is to be represented by the artist, is made of finite surfaces out there in the world. The painter's job is to find the appropriate shapes, sizes and positions for the counterparts of these surfaces within the microcosm of the painting in such a way as to constitute an *istoria*.

The totality of surfaces in the macrocosm exists objectively, though it changes from moment to moment with changes in the ambient light. (It is as if the sun, by a sort of divine fiat, makes a selection of which surfaces shall belong to this totality from moment to moment.) In addition to this global selection, however, each observer effects his own local selection from this totality in such a way as to yield a framed arrangement of observed surfaces of the sort which we see when we look through an open window. The array which results through this local selection is dependent upon the observer's position and on the scope and direction of his gaze. Moreover, some surfaces in the observed array are foreshortened because they have parts which are obstructed for this observer or are such as to fall outside the scope of what the artist will choose to represent. In this way there is created out of an in principle infinite totality a selection of a sort that can be comprehended by a finite mind.

[4] Alberti 1435–36, p. 89. Compare Leonardo's *Non mi legga chi non e matematico*. ('Let no one read me who is not a mathematician.')

4 Qualitative Geometry

For all of this, however, the results of this act of selection are, because they fall within the first kind of matter, still something entirely objective: they belong to the world of space and light out there. Compare the ontological status of the events which take place on the stage in a theater. Certainly the latter constitute a *play* only because of the way they are perceived and understood (and separated off by fiat from the events around them); but they exist nonetheless objectively, as movements of bodies and props. These movements are however subject to a further series of effects because of the ways the spectators in the theater react towards them. They find one movement threatening, another welcoming, and so on. And so also in our present case: the objective array of surfaces is subjected, when viewed by an observer, to effects of an analogous sort. Some surfaces will appear to be larger or of a different color or shape, some figures will dominate, others will recede into the background.

The artist's job, according to Alberti, is to project the objective array of surfaces into the microcosm of the painting in such a way as to achieve a maximally beneficial (moral) effect. The buildings, too, in which the painting is to be displayed, should likewise be designed on the basis of a combination of geometrical and moral principles, and the same applies also (Alberti was a pioneer of urban planning) to the city in which these buildings are to be arranged. (Westfall 1974)

Alberti is sometimes described as the first universal genius, and his work, whether on painting, on architecture, on town planning, or on the morality of the family, always transcends the purely theoretical sphere. This is no less true in the domain of mathematics, where Alberti was the first to present the geometrical principles of linear perspective. For even here his concerns point always in the direction of practical implications. As he himself expresses it: mathematicians examine the form of things as separated from their matter. Those, however, who wish the object to be seen 'will use a more sensate wisdom'. (1435–36, p. 42) Alberti's interest is accordingly not in form separated from matter, but rather in form as it is visible, which means: in the matter that is located in space and that is affected by ambient light.

He thus develops a version of Euclid's geometry not in terms of abstractions but in terms of concrete visible 'signs' or 'marks' (recall that the term used by Euclid himself for what we call 'point' is '*semeion*' or 'sign'):

> The first thing to know is that a point is a sign [*signum*] which one might say is not divisible into parts. I call a sign anything which exists on a surface so that it is visible to the eye. … Points joined together continuously in a row constitute a line. So for us a line will be a sign whose length can be divided into parts, but it will be so slender in width that it cannot be split … If many lines are joined closely together like threads in a cloth, they will create a surface. (Alberti 1435–36, p. 42)

It is in the same vein that Alberti proposes for the outer edge by which a surface is bounded the terminology of 'brim' [*ora*] or 'fringe' [*fimbria*], terms connoting the edge of a piece of cloth or garment. In a separate tract entitled *De punctis et lineis apud pictures* Alberti writes: 'Points and lines among painters are not as among mathematicians, [who think that] in a line there fall infinite points.' (Edgerton 1975, p. 81) Alberti hereby anticipates contemporary work on so-called qualitative geometries (Bennett *et al*., 2000), which means: geometries based, not on abstract mathematical points, but rather on finite regions. Both his theory of perspective and his theory of the organization of marks or signs to form an *istoria* are formulated in qualitative-geometrical terms.

5 Rays of Marvelous Subtlety

The surfaces in the objective array and their qualities of color, shape and size are, Alberti tells us, 'measured with sight'. What he means by this he explains by referring to 'the maxims of the philosophers' for whom there are rays that serve the sight 'which carry the form of the thing seen to the sense.' These visual rays, which are depicted in Figure 2 as extending between the single, fixed eye and the array of surfaces seen in the background, constitute what we have called a 'visual pyramid'. They are such that 'by a certain marvelous subtlety' they penetrate the air and 'all thin and clear objects' until

> they strike against something dense and opaque, where they strike with a point and adhere to the mark they make. Among the ancients there was no little dispute whether these rays come from the eye or the plane. This dispute is very difficult and is quite useless for us. It will not be considered. We can imagine those rays to be like the finest hairs of the head, or like a bundle, tightly bound within the eye where the sense of sight has its seat. The rays, gathered together within the eye, are like a stalk; the eye is like a bud which extends its shoots rapidly and in a straight line on the plane opposite. (Alberti 1435–36, pp. 44 f.)

Alberti's reference in this passage to 'the ancients' relates to the disputes among philosophers between so-called *intromissionist* and *extromissionist* views of visual perception. For the intromissionists, vision is to be explained in terms of light passing from the object and into the eye. For the extromissionists, vision is an active process involving 'visual rays', which move from the eye and out into the world of surfaces beyond. When Euclid, in his *Optics*, demonstrates theorems about visual angles, then it is in terms of an extromissionist theory of visual rays that these theorems are formulated. (The validity of the laws of geometrical optics work is not affected by the direction of the visual rays.)

For Euclid visual rays are homogeneous. For Ptolemy, on the other hand, another extromissionist, the centermost visual ray, which flows directly from the eye and strikes at right angles the surface of what is seen, is privileged as contrasted with 'median' rays on the fringes of the cone of rays emanating from the eye. When Galen isolated the eye's crystalline lens as the seat of visual power, he sees the lens, still in extromissionist terms, not as a receiver but rather as a transmitter of visual force.

Why, we might reasonably ask, did Euclid and Ptolemy and Galen, and many other prominent thinkers of the ancient world, defend what must nowadays seem so counterintuitive a view of visual perception? One reason was the supposed power of cats and other nocturnal animals to see in the dark. The primary argument for extromissionism however turned on the atomism embraced by many ancient thinkers. The extromissionists pointed out that it would be impossible that every point on a large visual surface should be transmitted simultaneously to a single point via atoms of light. The 'effluxes of things so large as, say, a camel or a mountain could not very well pass through the tiny pupil of the eye'. (Edgerton 1975, p. 67)

It was the Arab thinker Alhazen who, by solving this 'large efflux' problem, established the viability of intromissionist optics by showing how refraction can filter out excess information in the light. Alhazen showed how it was in fact possible even in atomistic terms for every point on the surface of an object seen in nature 'to convey its form to the seat of vision within the eye – in an exact one-for-one, place-for-place proportionate way.' (Edgerton 1975, p. 74) Alhazen's theory of refraction explained also the privileged status of rays close to the axis of sight: they travel unrefracted through to the optic nerve.

In his commentary on Aristotle's *De sensu*, Albertus Magnus distinguishes four positions on this dispute:

a. extromission of visual rays (for example Empedocles);
b. intromission of atoms (corporeal images: Democritus);
c. intromission of forms (spiritual images: Aristotle);
d. simultaneous extromission and intromission of rays (Plato);

Extromissionism lives on in the thinking of Augustine and Anselm, but the success of the intromissionist theory is given a powerful boost when it is taken up by three thirteenth-century English scholars: Robert Grosseteste, Roger Bacon and John Pecham, who saw in the new optical theories of the transmission of light the model of how God spreads the light of grace to his subjects in the world. As Veltman points out (1999), however, Leonardo could still defend a combined extromissionist-intromissionist view (and in presenting the arguments for an extromissionist component in the visual process Leonardo refers to the power which maidens have in their eyes 'to attract the love of men.') Even by the time of Kepler the debate was still not conclusively settled.

Indeed, extromissionism still lives on today. And this is so even in spite of all subsequent developments in our understanding of the physics of light and of the physiology of the eye. It lives on in contemporary philosophy and cognitive science in the context of discussions of what is nowadays called 'intentionality'.[5] And it is in these terms, I suggest, that the visual rays of Alberti – and perhaps even of Euclid[6] – are to be understood.

'Intentionality' is the term employed especially in the tradition of Brentano, Husserl and Ingarden to refer to the directedness of the mind towards its objects. Husserlians sometimes speak of the *arrow* of intentionality, and Husserl himself (1970) uses the terminology of 'mental rays' for example when he distinguishes between single-rayed and many-rayed acts of perception. All cognitive directedness towards objects, for Husserl, depends on perceptual intentionality and all perceptual intentionality depends on action. (Mulligan 1995) Language, including maps and diagrams, Husserl sees as a vehicle by which intentional directedness is leveraged beyond the realm of objects given in direct perception.

Gibson's ecological psychology, too, can be understood in these terms. It represents a mixed intromissionist-extromissionist view, according to which each organism, in each given context, is tuned to certain specific types of invariants within its surrounding ocean of energy. The organism picks up the information available in the environment that is relevant to its actions in a spontane-

[5] Compare also the extromissionist theory of vision propounded in Pylyshyn (1989), which presents a view of the visual system as employing a limited number of visual indexes that go out into the world and adhere to whatever it is that the visual system wants to interrogate. These indexes are hypothesized to allow access to the objects that they individuate, and because they are sticky, they are able to track objects and to be updated automatically as the object moves about in the environment.

[6] On this reading, the visual rays which Euclid conceives as projecting from the eye are not to be conceived in physical terms at all. For the theory of perspective belongs not to physical but rather to geometric optics, which is what results when we adopt simplifying assumptions to the effect that the wavelength of light is zero and that rays propagate through homogeneously refractive media along straight lines. Euclid's visual rays would then be analogous to the abstractly conceived (fiat) lines of his own geometry, rather than to rays of light in the proper, physical sense – or indeed to X-rays, or to any other *physical* manifestations of 'marvelous subtlety'.

ous process which involves, not inference or other cognitive processes, but rather (in our terminology) something like a transparent grid into the cells of which the affordances of the environment exactly fit.

The views presented here can now be seen as a generalization of the Husserlian and Gibsonian accounts of organism-environment interactions to apply also to the projection onto reality that is involved in our uses of maps, pictures, databases, catalogues and taxonomies of various sorts. When we use a proper name to refer to an object, then a relation of projection comes into existence: the name projects out towards the object, be it present or absent, in virtue of an intentional ray. When we use a photograph to refer to an object, then a relation of projection likewise comes into existence: the photograph projects out towards the object in virtue of a whole pattern of intentional rays. And similarly, when using maps or spreadsheets we employ labelled grids to project in multi-rayed fashion onto corresponding objects in reality.

H 1																	He 2
Li 3	Be 4											B 5	C 6	N 7	O 8	F 9	Ne 10
Na 11	Mg 12											Al 13	Si 14	P 15	S 16	Cl 17	Ar 18
K 19	Ca 20	Sc 21	Ti 22	V 23	Cr 24	Mn 25	Fe 26	Co 27	Ni 28	Cu 29	Zn 30	Ga 31	Ge 32	As 33	Se 34	Br 35	Kr 36
Rb 37	Sr 38	Y 39	Zr 40	Nb 41	Mo 42	Tc 43	Ru 44	Rh 45	Pd 46	Ag 47	Cd 48	In 49	Sn 50	Sb 51	Te 52	I 53	Xe 54
Cs 55	Ba 56	La 57	Hf 72	Ta 73	W 74	Re 75	Os 76	Ir 77	Pt 78	**Au** 79	Hg 80	Tl 81	Pb 82	Bi 83	Po 84	At 85	Rn 86
Fr 87	Ra 88	Ac 89	Rf 104	Ha 105	?? 106												
Lanthinide Series		Ce 58	Pr 59	Nd 60	Pm 61	Sm 62	Eu 63	Gd 64	Tb 65	Dy 66	Ho 67	Er 68	Tm 69	Yb 70	Lu 71		
Actinide Series		Th 90	Pa 91	U 92	Np 93	Pu 94	Am 95	Cm 96	Bk 97	Cf 98	Es 99	Fm 100	Md 101	No 102	Lr 103		

Fig. 3 The Periodic Table of the Chemical Elements

6 How to Tell the Truth with Maps

A good map casts a transparent net over the surface of the earth in just the way Alberti's *reticolato* casts a transparent net over some portion of objective reality. As the painter's grid casts into relief a certain visual scene, so the grid of the map casts into relief a certain spatial region. There is a deep-seated analogy here; but it is an analogy *that has nothing to do with perspective* – for it obtains even in relation to maps and plans of strictly two-dimensional planar arrays. It has to do, rather, with the highly general concept of a *transparent grid* and with the associated highly general notion of *projection*, both of them notions which (as Figure 3 makes clear) can be applied even to types of organization which are entirely non-spatial. (Bittner and Smith, in this volume.)

To see what all of these cases have in common we need to boil Alberti's *reticolato* down to its essential elements, which we can list as follows:

1. the eye (or point of projection),
2. projective rays,
3. the artist's grid,
4. the constituent cells of the artist's grid,
5. the totality of objective visible surfaces,
6. the target grid: the artist's grid as projected onto the objective visible surfaces,
7. the constituent cells of the target grid

Extending from the point of projection (1.), projective rays (2.) bring about a one-one correspondence between the two arrays of cells (4. and 7.), within the artist's grid (3.) and the target grid (6.), respectively. We shall call a structure of this sort a *true grid*. A true grid is transparent to the corresponding objects in reality.

Almost all our customary maps are true grids in the sense defined. In this case the term for the eye or station-point (1.) is replaced by that of the user of the map. The projective rays (2.) are replaced by relations of rigid designation (to be discussed below). The counterpart of the artist's grid (3.) is constituted not just by the rectilinear grid of the map but more generally by whatever is the pattern of contour and border lines and cartographic icons (4.) to be found on the map. The counterpart of (5.) is the corresponding portion of the earth's surface, and the counterparts of (6.) and (7.) are the results of projecting the grid of the map onto this more or less planar region.

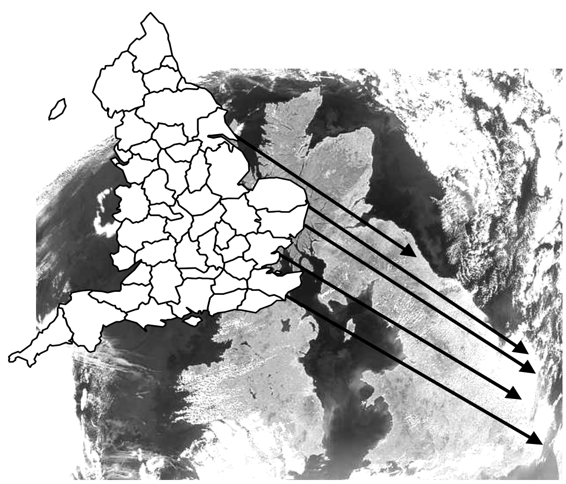

Fig. 4 Cartographic Projection

As in the *reticolato*, so also here we need to distinguish in the ontological structure of the map between two distinct grids. On the one hand is the grid of the map itself, which is in the simplest case a system of regular or irregular cells, each cell enjoying a certain intrinsic position within the grid and thus also standing in certain determinate relations to its neighboring cells. On the other hand is an isomorphic grid on the side of the target portion of the surface of the earth (in Figure 4 a grid of English counties). Regular cells in such grids will standardly be identified by their coordinates within the grid itself; irregular cells will standardly be assigned proper or common noun labels such as ‘Berkshire’ or ‘lateral geniculate nucleus’.

In the case of the *reticolato*, the projective rays can be made to point in whichever direction the user might desire. In the case of a map, on the other hand, the projective rays tie the cells of the map rigidly to corresponding portions of reality. This means that when a user buys a map, he buys not any simple piece of paper but rather a complex cognitive device from out of which, as soon as he begins to use the map, there will project invisible arrows – rays of marvelous subtlety – which tie its constituent cells rigidly to corresponding features on the ground.[7]

7 Semantic Projection

Such lines of projection are at the basis, too, of the so-called picture theory of meaning defended by Wittgenstein in the *Tractatus*:

> The pictorial relationship consists of the assignments of the picture’s elements to the things.
> These correlations are, as it were, *the feelers of the picture’s elements, with which the picture touches reality*. (2.1514 f., italics added)

A true proposition, for Wittgenstein, is a picture or map of a state of affairs in reality. It is a propositional sign *in its projective relation to the world*. (3.12) Each (atomic) proposition in the Tractarian framework consists of simple signs (names), which stand in a projection relation to corresponding simple objects. If the proposition is true, then these simple signs stand to each other in the propositional picture as the corresponding objects stand to each other in the world. Here the counterpart of the artist’s grid, in Alberti’s terminology, is the propositional sign, a complex of names arranged in a certain order, the names serving as the equivalent of the constituent cells. The counterpart of the target grid is a state of affairs in the world.

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[7] We can now see our way to resolving a thorny problem highlighted by Ernst Gombrich, which turns on the apparent irreversibility of the projective function involved in maps and pictures. The theory of perspective representation, as Gombrich notes,

> was treated as if it were a mapping procedure. It was claimed that it enabled the artist to represent what has been called ‘measurable space’. Yet … it is clear from the theory of central projection that you cannot reverse the process: while we can work out what the projection of a three-dimensional object will be like on a given plane, the projection itself does not give us adequate information about the object concerned, since not one but an infinite number of related configurations would result in the same image, just as not one but an infinite number of related objects would cast the same shadow if placed in the beam emanating from a one-point source. (1974, pp. 190 f.)

Gombrich extends this indeterminacy of projection also to maps: the answer to the question ‘how can we ever know whether a picture or a map represents a particular building?’ is, he says, simple: *we cannot*. (Op. cit., p. 175) But surely something has gone wrong with Gombrich’s thinking here. Millions of people are, after all, using maps perfectly successfully every day to find particular token buildings. To do justice to this fact we need to recognize that the lines of projection emanating from maps, as also from representational paintings, and photographs and other similar projective devices are not purely geometrical. Rather, they involve also a combination of semantic, perceptual and other projective elements and some of their constituent cells are analogous to proper names, which tie their users rigidly to specific object tokens.

It is a complex question how far Wittgenstein's picture theory of meaning can be extended to language in general. Where it does unproblematically apply is in relation to simple lists, which constitute true grids in the sense here intended – provided only that the items listed do indeed exist in reality. Even a single name, for example 'Mama', constitutes a true grid under the obvious ('Mama' – Mama) projective relation.

Moreover, as Figure 3 once more reveals, a system of concepts, too, can form a true grid in the sense defined. The idea of a projective relation from concepts to corresponding categories on the side of target objects in reality is at work in the following passage from Millikan:

> The membership of the category 'cat,' like that of 'Mama,' is a natural unit in nature, to which the concept cat *does something like pointing*, and continues to point despite large changes in the properties the thinker represents the unit as having. ... *The difficulty is to cash in the metaphor of 'pointing' in this context.* (Millikan 1998)

The generalized *reticolato* and the associated notion of projection can, I suggest, help to cash in Millikan's pointing metaphor in precisely the way required.

The generalization from Alberti's *reticolato* to maps is in one sense simple. Both involve grids which are recognizable as such; both involve relations between spatial neighborhoods which can easily be defined in topological terms; both types of grid can also be subject to the same types of geometrical transformations. It is at first sight difficult to see how we can generalize beyond these sorts of cases to talk of semantic or conceptual grids. In light of recent advances in mereotopology and in the study of so-called 'conceptual neighborhoods' or 'continuity networks', and also in light of the work on granular partitions outlined in the paper by Bittner and Smith (in this volume), we can more readily understand what such generalized grids involve. They all share in common – in the ideal case – the presence of a domain (the user's grid) and a co-domain (the target grid), with systems of mereotopological relations defined on each, and with a notion of correspondence or mapping connecting the two of a sort which – in the case of a true grid – preserves mereotopology.

But not all grids are true. For while the examples dealt with so far have involved an isomorphism between cells in the grid and corresponding objects, grids may fall short of such perfection by involving some mismatch between user's grid and target grid. Such a mismatch can come about either because the projective relation is not well-defined (cells in the grid are putatively projected onto objects where there are no such objects) or because the cells of the grid do not stand to each other in relations isomorphic to the relations between the corresponding target objects.

Even grids which satisfy both of these requirements may still fall short of the sort of perfection that is manifested in the examples of optical, cartographic and conceptual projection referred to above. The grids of the latter satisfy a requirement to the effect that the cells within the target grid fit exactly to the corresponding cells of the user's grid. This condition can in various ways be weakened. Bittner and Stell (1998) offer an approach to spatial grids otherwise similar to the one advanced here but within which the restriction on cell-object fit is relaxed through the notion of 'rough' location. Smith and Brogaard (2001, 2001a) show how the theory of true grids can be used to develop a new version of the supervaluationist theory of vagueness. Their work turns on the idea that grids are always such as to involve a certain coarse-grainedness, which implies that their cells trace over parts or features of reality which fall beneath a certain size. This in turn means that the latter can vary while the user's grid – which represents our cognitive access to the relevant objects – remains the same. The phenomenon of vagueness, from this perspective, is just the other side of the coin from the phenomenon of granularity. It arises because of the possibility of a variation that falls, in a given context, beneath the threshold of salience.

8 Directions of Fit

Each true grid – be it optical, cartographic, or conceptual – effects a tiling of the portion of reality towards which it is directed. In some cases, as in the case of a gridded map or an Albertian grill, this imputed tiling – a system of fiat boundaries in reality to which the grid directly corresponds – is of no intrinsic significance. Even a purely arbitrary imputed tiling may, however, acquire significance if its fiat cells are put to specific practical purposes by colonial administrators or postal authorities. (Smith 2001) In some cases, however, the grid of a map reflects prior independently existing boundaries on the side of the objects themselves. This is so, for example, of the irregular grid depicted in Figure 4 above, which reflects not only the fiat division of England into counties but also the bona fide division between England and the sea, which (partially) surrounds it. In yet other cases – this is so above all in the case of cadastral maps – the grid of a map stands to its target grid in a symbiotic relationship. As the objects change (because the fiat boundaries of land parcels are redrawn or re-measured, or because the land itself has been subject to erosion), so corresponding changes are made in the grid of the cadastre; and as administratively motivated changes in the grid of the cadastre are effected, so this may bring about changes in the objects (land parcels) on the ground.

We can thus distinguish, for true maps, three different sorts of cases:

1. the target grid depends exclusively on the grid of the map (a map-to-world direction of fit)
2. map grid and target grid are mutually dependent upon each other (as in the case of a cadastre)
3. the grid on the map reflects a pre-existing grid in reality (a world-to-map direction of fit).

3. can be divided into further sub-cases, according to whether the grid of the map reflects

3a. bona fide boundaries on the side of the target objects themselves
3b. pre-existing fiat boundaries on the side of the target objects
3c. some combination of bona fide and pre-existing fiat boundaries.

The same family of cases can be distinguished also in the domain of conceptual projection. Corresponding to 1. is the case where the distinctions in the world are mere reflections of our concepts (for example when the baseball coach divides up his team by assigning positions to his players at the beginning of the game). An example under 2., the symbiotic case, might be the set of prize categories used by dog shows, which both reflects the divisions on the side of the sample domain and also, over time, may itself bring about adjustments to these divisions. Corresponding to 3., finally, is the case where concepts reflect pre-existing distinctions among objects in reality, whether bona fide (3a.), for example the distinctions between the six different sorts of quark; or fiat (3b.), for example distinctions between different tax brackets; or mixed (3c.), for example the distinctions among bird species or among languages and dialects.

9 Windowless Monads

Epistemological skepticism is a view to the effect that conceptual classifications of type 3a. are forever beyond our reach. Such epistemological skepticism goes hand in hand with the views of many artists in recent times, who have been pleased to ignore perspective geometry, just as they have ignored Alberti's four principles of *dignità*, *varietà*, *modestia* and *verisimilitudo*. From at least the time of Duchamp, the visual arts have been freed from their connection to everyday life (and to beauty and harmony) and they have been recontextualized in the museum. The function of painting, if it has one, is not at all that of providing a window on the world, but rather that of drawing attention to itself. A painting is no longer conceived as a transparent device enabling the perceiver to grasp the reality beyond. Rather, it is an object in its own right, and the viewer is called upon to relish its materiality and its quality of opaqueness.

Talk of a 'correct' perspectival representation, with its implication to the effect that there is some single detached master point of view, has at the same time come to be disparaged as a remnant of outmoded phallologocentric thinking. How can one or another method of painting be 'true' or 'correct', when there is no single notion of reality against which its results could be matched? As Henri Lefebvre puts it in his *Production of Space*:

> The fact is that around 1910 a certain space was shattered. It was a space of common sense, of knowledge (*savoir*), of social practice, or political power ... a space, too, of classical perspective and geometry, developed from the Renaissance onwards on the basis of the Greek tradition (Euclid, logic) and bodied forth in Western art and philosophy, as in the form of the city and town. (1974, p. 25)

There is a simple argument from the realist side against all such nonsense. It is the same argument which can be used against all attempts to see reality as somehow dependent upon people's beliefs, and against all attempts to identify scientific truths with mere conventions of time or culture. In our present case the argument would run as follows: if perspective geometry is not inherent in the world – a structure waiting to be *discovered* – but rather a convention, which had to be *invented*, then this has the consequence that Renaissance men were *living in a different world* from the world of their medieval predecessors. It is this consequence which makes the skeptical and anti-realist positions seem so glamorous and exciting. But it is evidently a consequence no less absurd than the thesis that, with the dawning of the realization that the earth is round, the earth itself acquired a new geometry. To this, characteristically, the defender of the anti-realist view will respond that of course he is not wishing to be taken literally when he says that Renaissance men were *living in a different world*, or that they were '*producing*' or '*shattering*' *a certain space.* Such remarks are, he will say, mere metaphors. But then surely the interesting questions pertain, not to what metaphors have been fashionable at different points in human history, but rather to what the true structure of reality is – and this is a question which makes sense only from the realist perspective.

As the physiologist M. H. Pirenne (1952) shows, even granting the simplifying assumptions of geometrical optics, perspective paintings correspond to the way we see the world around us with a very high degree of approximation. The best explanation of this correspondence lies in the thesis that the mathematical forms captured in the geometry of perspective are – modulo certain well-understood and in normal circumstances negligible simplifications – out there in the world, waiting for us to apprehend them through abstraction. They thus serve as truthmakers for the theory of perspective; and as Pirenne nicely puts it, the strange fascination which perspective had for the Renaissance mind was precisely 'the fascination of truth.'

Certainly our *understanding* of perspective has developed over time. For whatever reason, it took a long time before people were ready to perform the abstraction of these mathematical forms. We can hazard that part of this reason turns on the need, before this step could be taken, for a certain detachment from the world of objects through the cultivation of the standpoint of the neutral, scientific observer, a standpoint which Renaissance thinkers, like some of their Greek predecessors, enjoyed, but which medieval thinkers lacked. Renaissance thinkers such as Alberti were able to grasp the world as an abstract, mathematical 'container' – as a stage upon which men move, and have their exits and entrances. It is indeed in the Renaissance that the theatrical audience is for the first time separated from and forced to adopt a particular point of view (or as we might also say, a particular perspective) in relation to the spectacle on the stage.

10 Fit Happens

There is nothing subjective in Alberti's *reticolato.* As Pirenne makes clear, the geometrical relationship between an object and its projection on the picture plane obtains quite independently of whether there is an eye at the vanishing point. As the technology of laser-guided missiles reveals, the laws of perspective hold independently of the existence of subjects, observers, artists or cultures: they are laws governing the way light, space and the surfaces of objects are related together. The laws of perspective are laws of geometrical optics; they have nothing to do with neurology or psychology. Correspondingly, the picture drawn in perspective aims not at representing anything like the retinal image or any pattern of nervous stimulation on the side of an observer. Rather it aims to send to the eye the same distribution of light as that which the object itself would send.

This corresponds to the theory of picture perception sketched by the great theorist of realism J. J. Gibson and encapsulated by his student Kennedy (1974) in the form of a definition of a picture as: 'a surface treated so that it yields light to a particular station point, usually on a normal to the picture surface, which could have come from a scene in the real world.' (Compare Gibson 1978.)

Gibson naturally recognizes that there are other sorts of pictures (including maps), some of which involve conventional elements (symbols, icons), which have nothing to do with the conveyance of light to the eye in a way which simulates the light that is projected from surfaces in three-dimensional space. Even these pictures can, however, be interpreted in realist fashion on the basis of the general theory of projection sketched above. There are of course also many cases of pictorial images in which perspectival or other features of the represented scene are distorted in one or other way. As the Gibsonian realist can insist, however, the fact that pictures are sometimes made, for whatever reason, in such a way as to embody such distortions does not imply that *all* pictures are lacking in the sort of transparency for which the followers of Alberti strove.

All maps must be of a certain scale or combination of scales, just as every grid must have a certain resolution or granularity of cells. And since reality itself (as Gibson 1979 emphasizes) contains entities accessible at many different scales, it follows that no single grid can be complete. Rather, as scientific practice shows, we need grids of many different resolutions if we are to do justice to reality in its many aspects. This implies, as the enemies of realism are fond of pointing out, that there is no 'God's eye perspective' or 'view from nowhere'. This does not, however, mean that we are justified in drawing the conclusion that every single one of the myriad perspectives which we have at our disposal embodies a false view of reality. The inference from partiality to falsehood might indeed be valid, but only in a world without windows – a world in which no single one of our grids enjoys the condition of transparency.

The fact that there are maps which deviate, for whatever reason, from the strictly veridical representation of reality does not take away from the fact that – leaving aside any small errors which may have been made in the application of the relevant projection system – almost all maps are *true of* the corresponding portion of reality. This applies to Mercator's map, and it even applies to Saul Steinberg's *View of the World from Ninth Avenue*. Maps must of course embody some projection system in representing three dimensions on a planar surface. Yet those who see in this an argument to the effect that all maps must necessarily involve some form of systematic distortion are simply revealing their own misunderstanding of the nature of projection. They are like those who, on noticing that the Circle Line is represented on maps of the London Underground as a yellow band, complain of 'distortion' because yellow, rather than some other color, has been used.

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