Computer Graphics and Image Processing

(a) Homogeneous coordinates are often used to represent transformations in 3D:

$$\begin{bmatrix} x'_H \\ y'_H \\ z'_H \\ w'_H \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \\ c_1 & c_2 & c_3 & d \end{bmatrix} \begin{bmatrix} x_H \\ y_H \\ z_H \\ w_H \end{bmatrix}$$

(i) Explain how to convert standard 3D coordinates, (x, y, z), to homogeneous coordinates, and how to convert homogeneous coordinates to standard 3D coordinates. [2 marks]

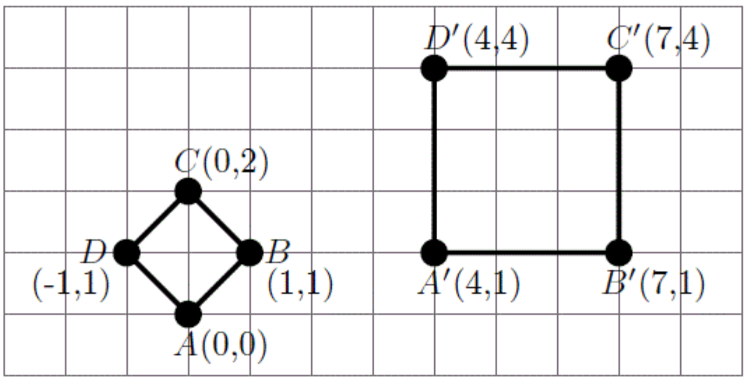
(ii) Describe the types of transformations provided by each of the four blocks of coefficients in the matrix ($a_{11} \ldots a_{33}$, $b_1 \ldots b_3$, $c_1 \ldots c_3$ and d). [5 marks]

(iii) Explain what transformation is produced by each of the following matrices:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & p & -p(1+r) \\ 0 & 1 & q & -q(1+r) \\ 0 & 0 & 1+r & -r(1+r) \\ 0 & 0 & 1 & -r \end{bmatrix}$$

[5 marks]

(b) Consider the following figure:



(i) Give a matrix, or product of matrices, that will transform the square $ABCD$ into the rectangle $A'B'C'D'$. [5 marks]

(ii) Show what happens if the same transformation is applied to $A'B'C'D'$. [3 marks]