



# Computing Hard Contacts with heterogeneous Materials for Medical Simulators

Maximilian Kaluschke

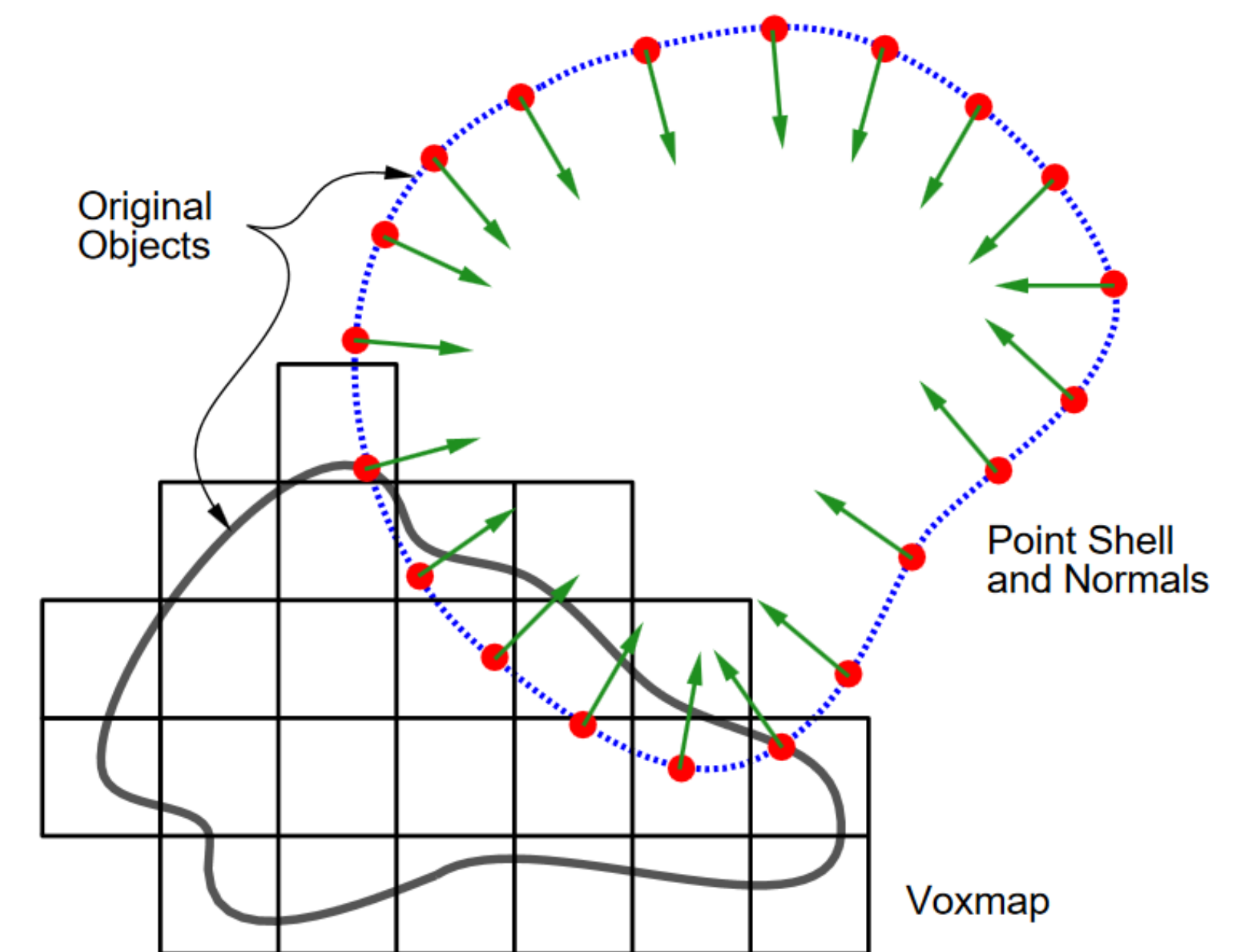
# Introduction

- Arthritis: 9% of 20+ y/o  $\rightarrow$  >90% of 65+ y/o
- Hip replacement
  - Prevalence
    - >140.000 procedures in 2016 in Germany
    - Cost-effective quality of life improvement
  - Hip socket milling is difficult to learn
  - Surgeon's skill major factor

# Introduction



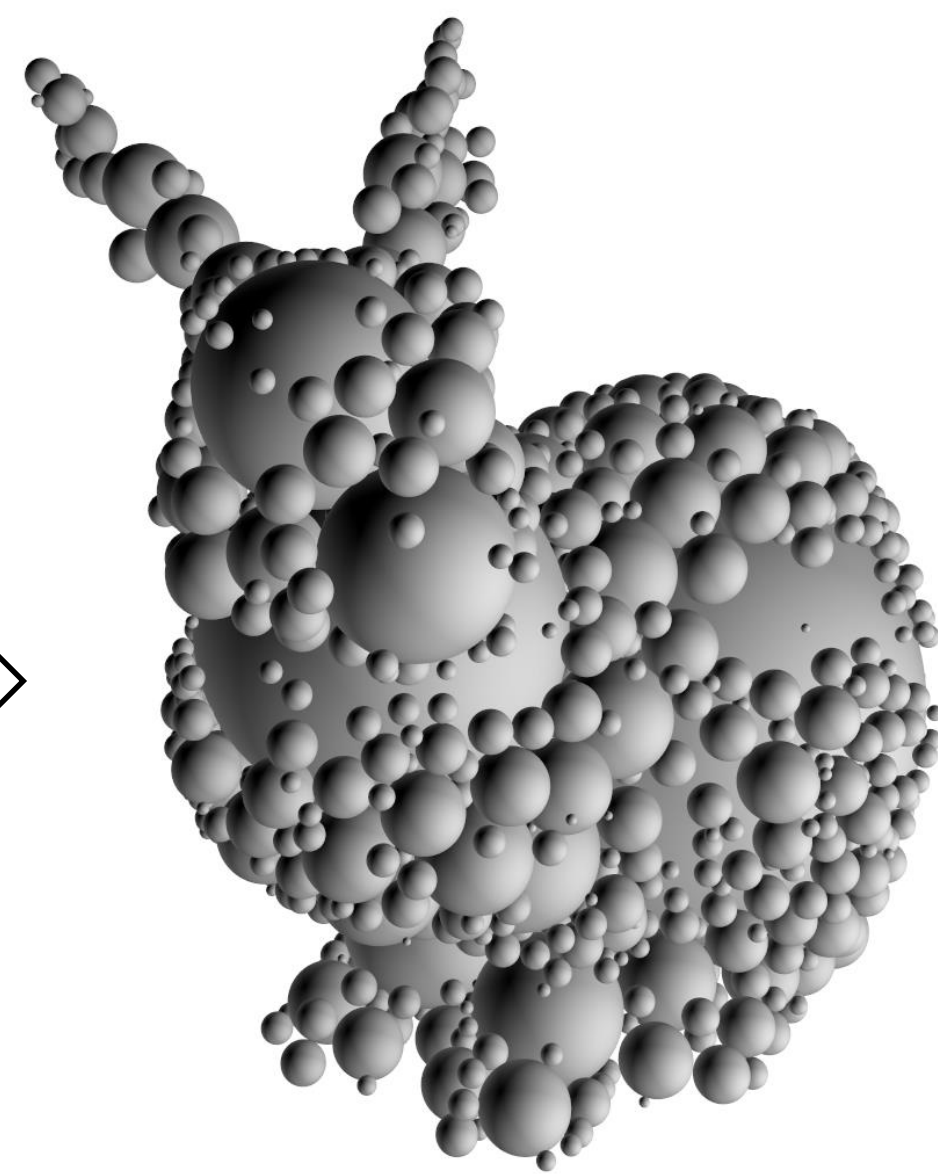
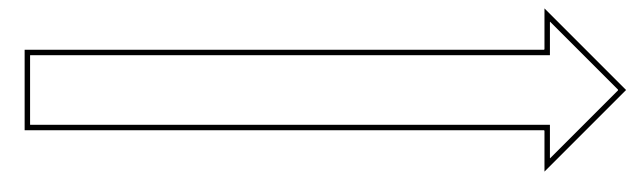
- Six Degree-of-Freedom Haptic Rendering Using Voxel Sampling;  
*McNeely, 1999*
- A GPU-implemented physics-based haptic simulator of tooth drilling; *Razavi, 2015*
- Inner sphere trees and their application to collision detection; *Weller, 2008*



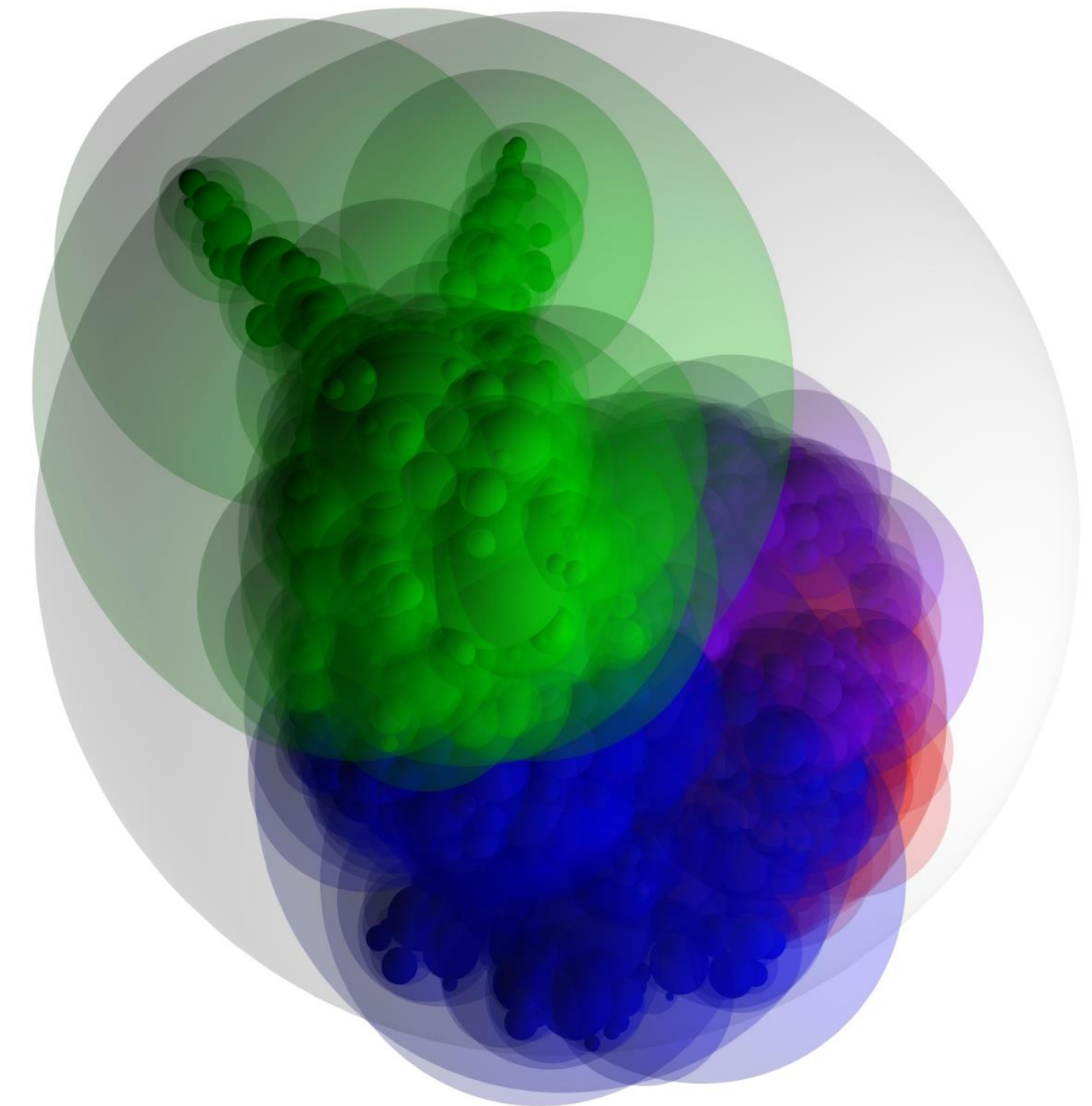
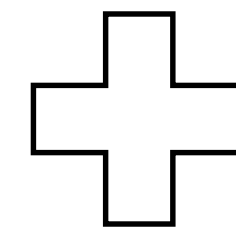




Container



Inner spheres



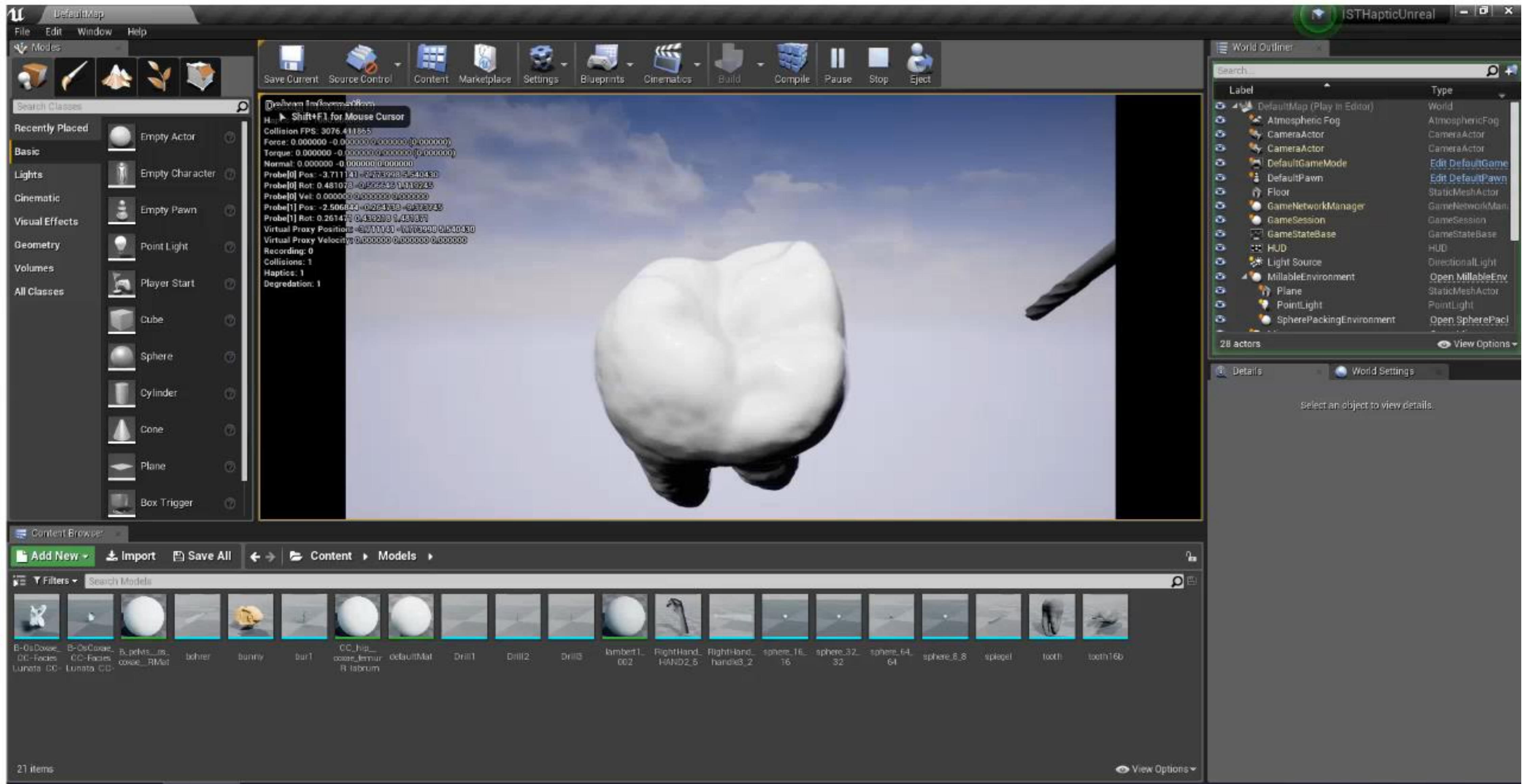
Bounding volume hierarchy



# Teaser

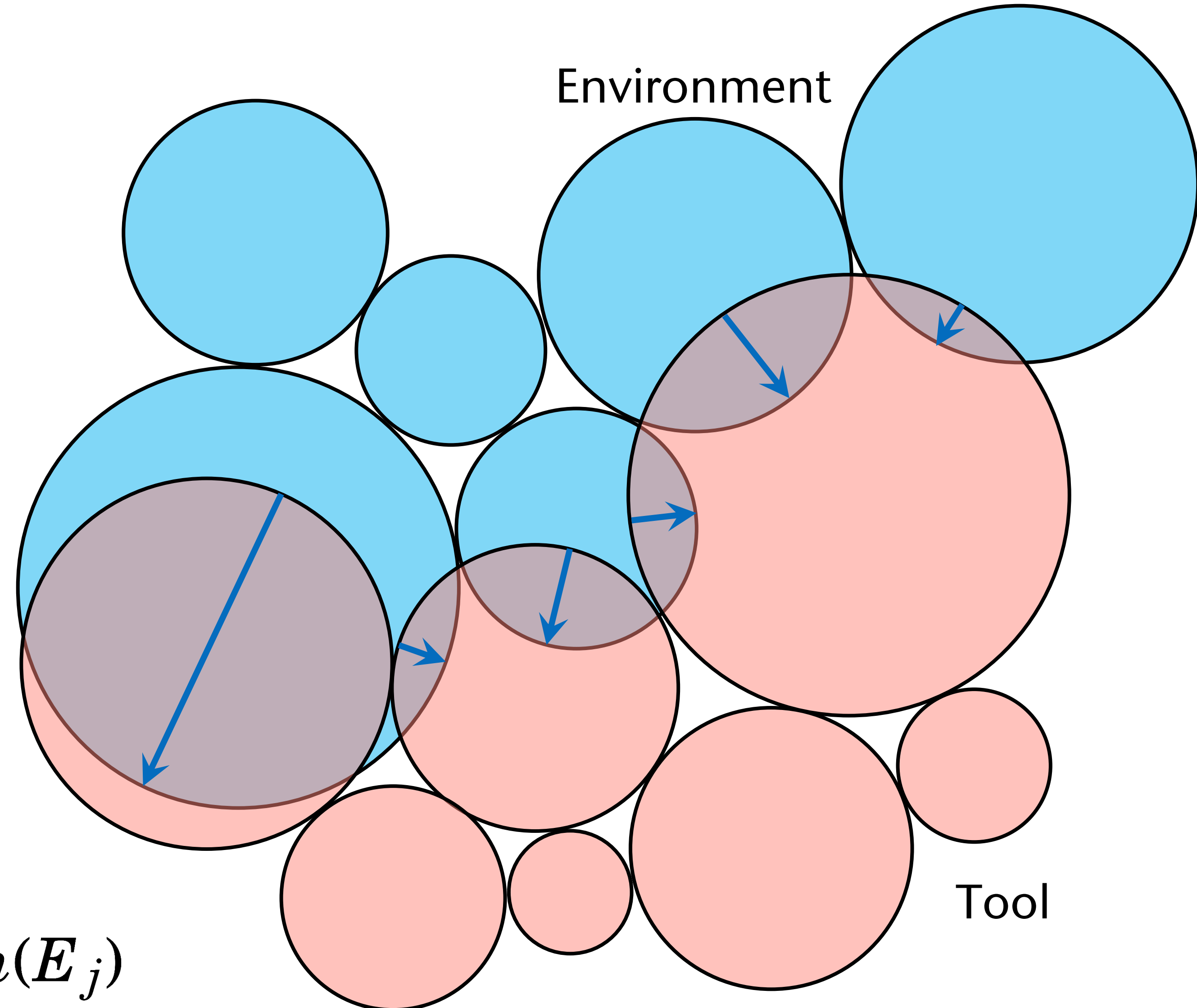






# Collision Detection — Force Computation

- Penalty-based forces
  - Overlap defines direction and magnitude of force
  - Easy to implement
  - Good performance

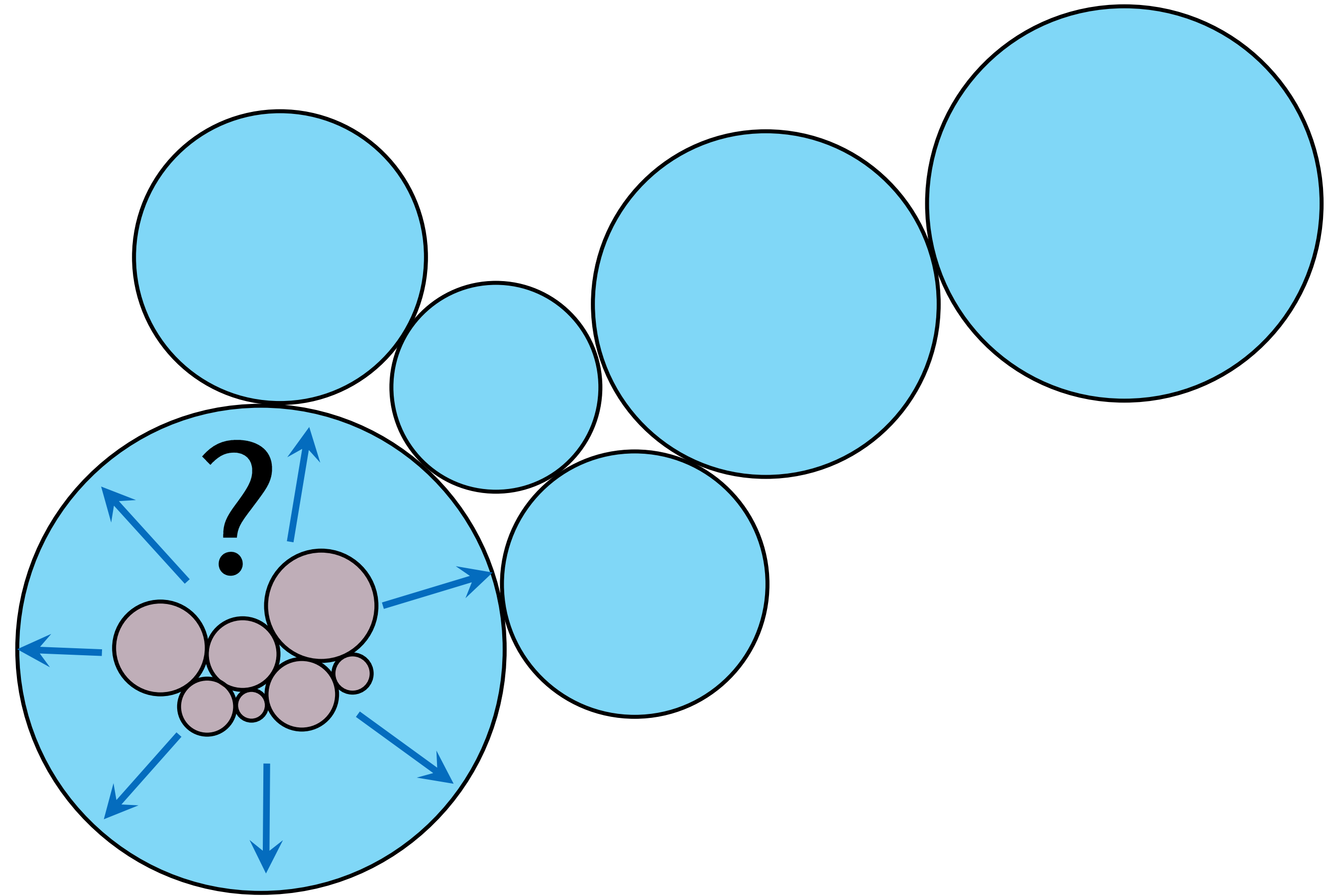


$$F_p = k \sum_{T_i \cap E_j \neq \emptyset} \text{Vol}(T_i \cap E_j) \cdot n(E_j)$$



# Collision Detection — Force Computation

- Penalty-based forces
  - Overlap defines direction and magnitude of force
  - Easy to implement
  - Good performance
  - Introduces potential problems
    - Solution: Normal cones

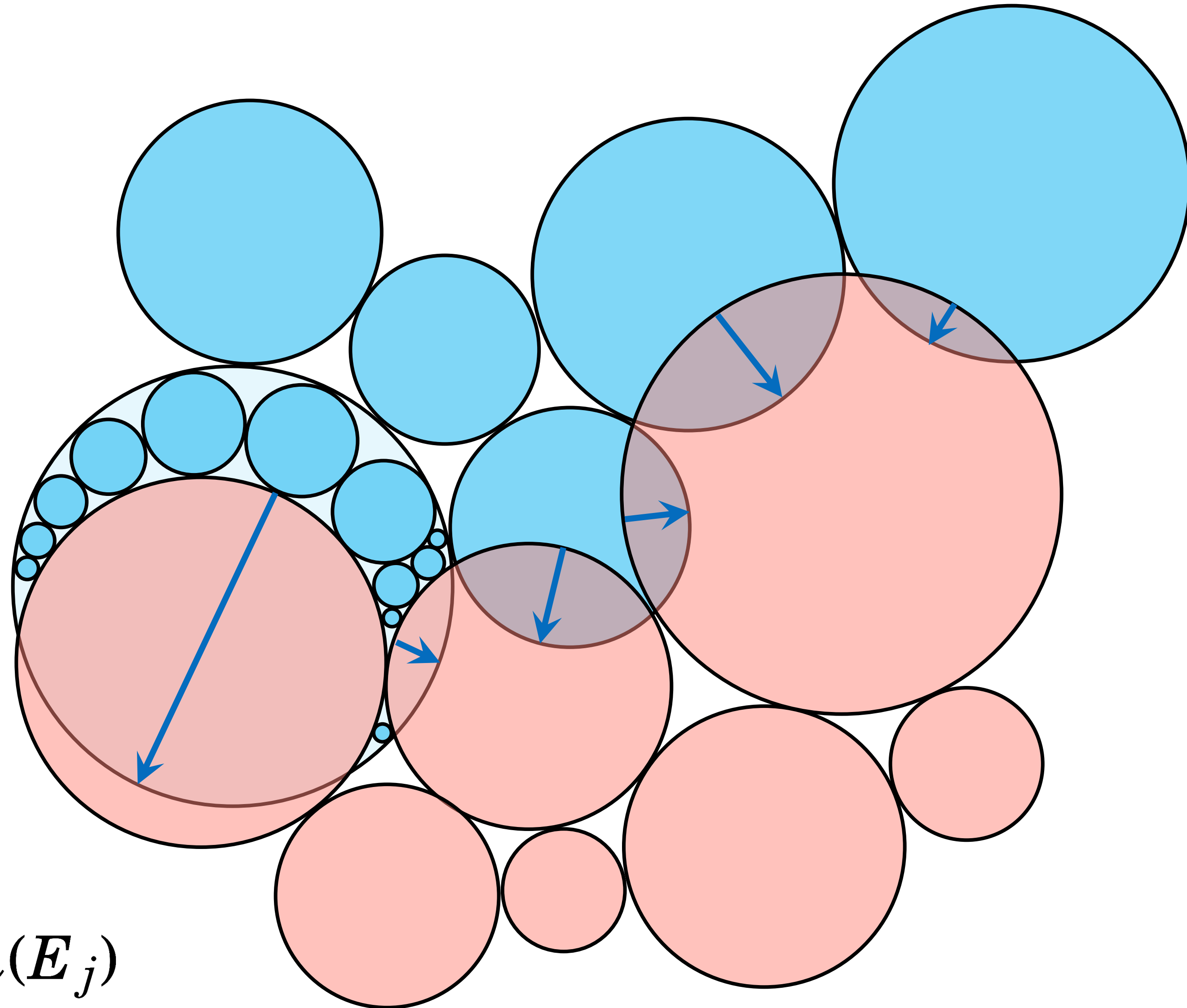


$$F_p = k \sum_{T_i \cap E_j \neq \emptyset} \text{Vol}(T_i \cap E_j) \cdot n(E_j)$$

# Collision Detection — Force Computation

- Penalty-based forces
  - Overlap defines direction and magnitude of force
  - Easy to implement
  - Good performance
  - Introduces potential problems
    - Solution: Normal cones
  - Discrete drill simulation

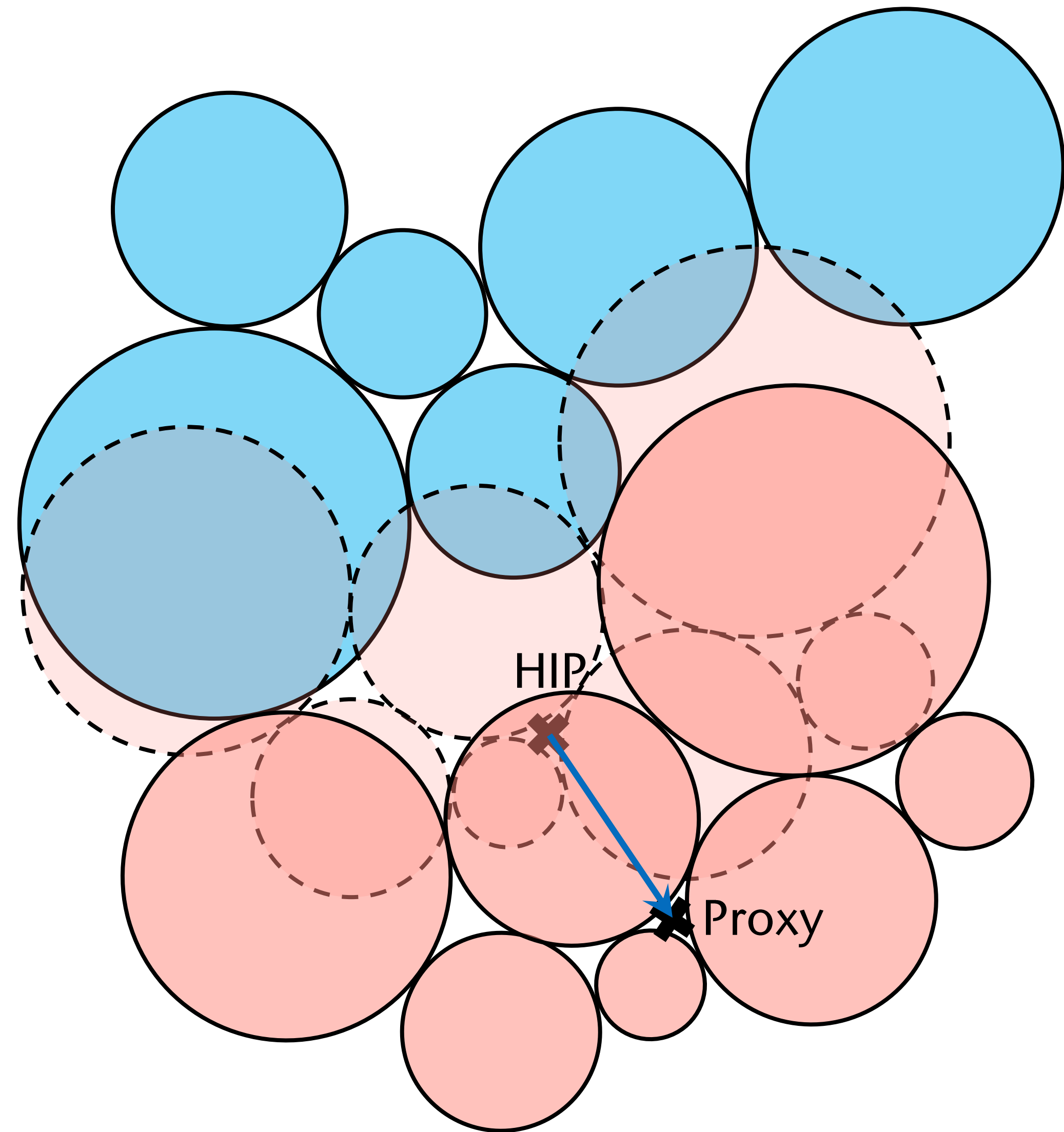
$$F_p = k \sum_{T_i \cap E_j \neq \emptyset} \text{Vol}(T_i \cap E_j) \cdot n(E_j)$$



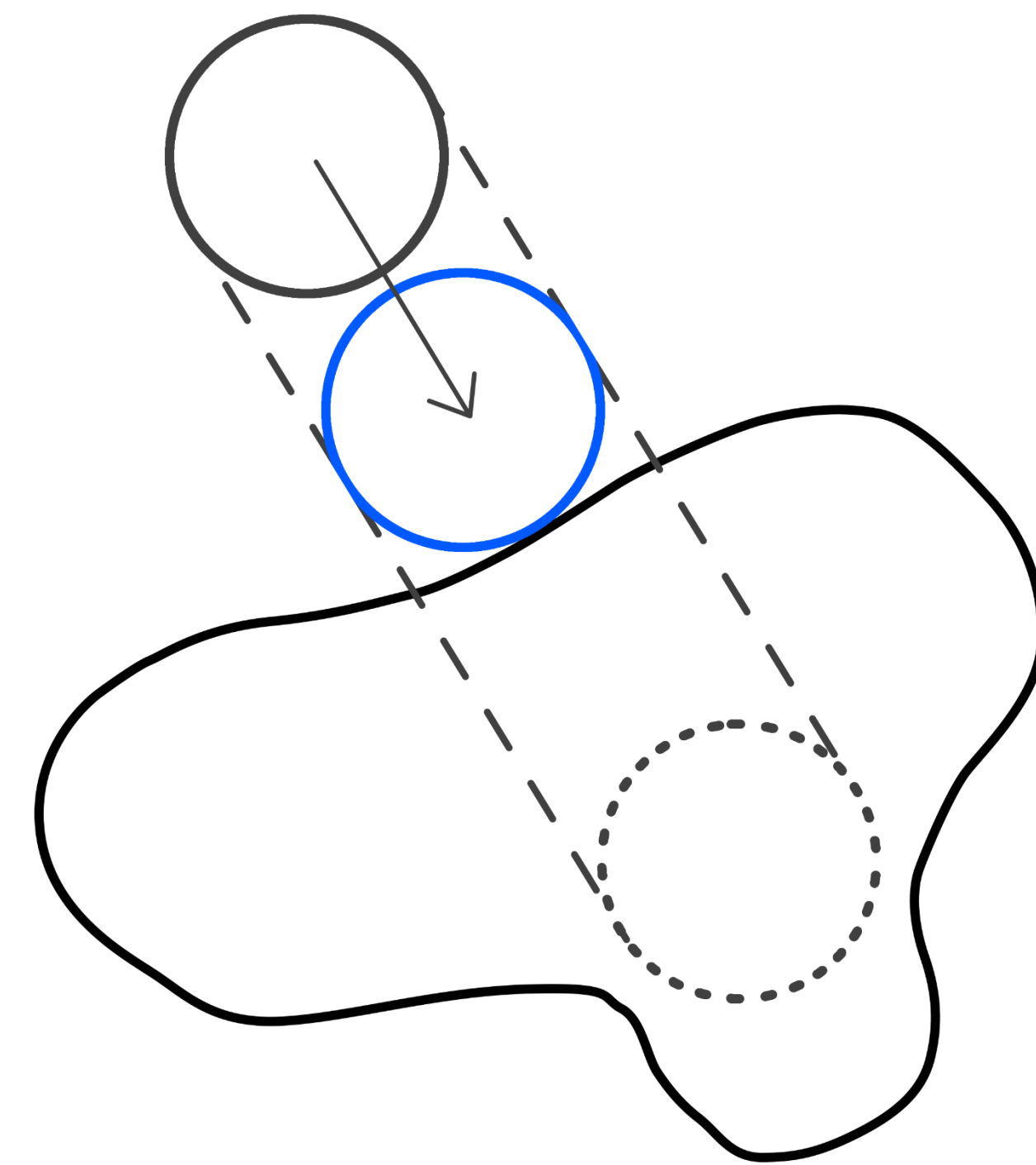


- Constraint-based forces
  - Well-defined force direction (at all times)
  - Extra instance of tool (called „proxy“)
    - Constraint by environment surface
  - Not *much* harder than penalty method for 3 DoF

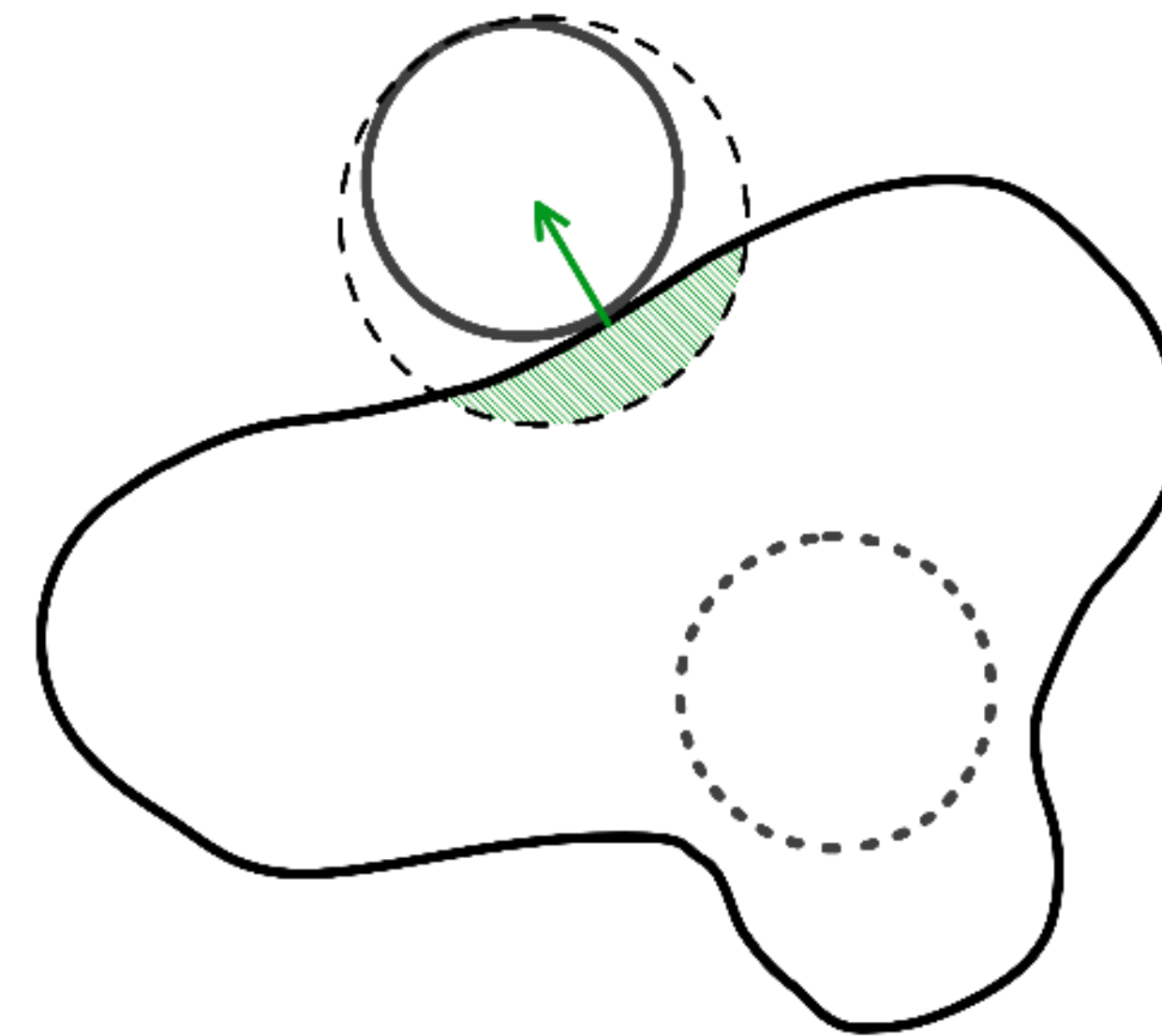
$$F_c = k \cdot (P_{Proxy} - P_{HIP})$$



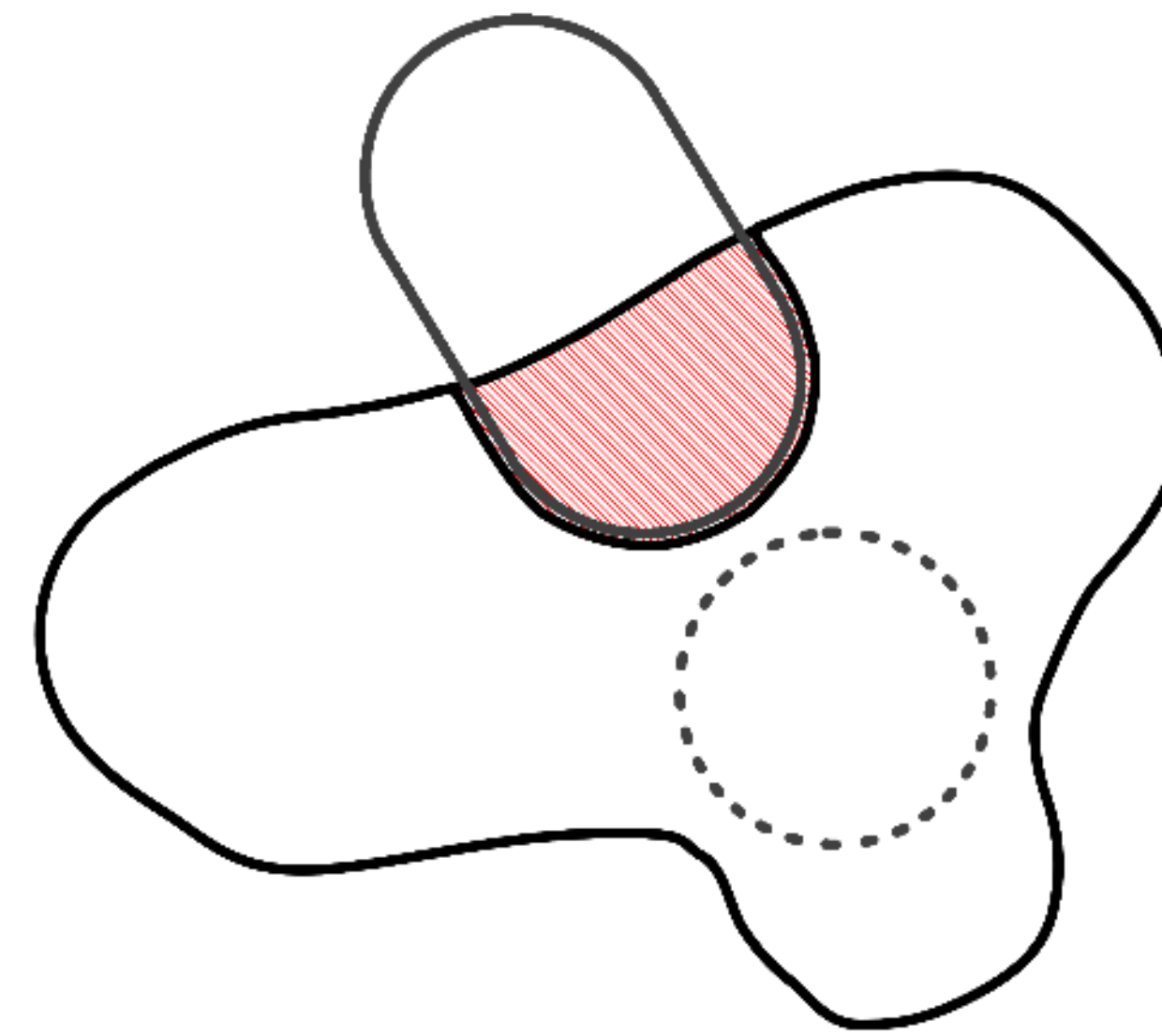
# Simulation — Overview



1. Surface contact



2. Surface estimation



3. Drilling

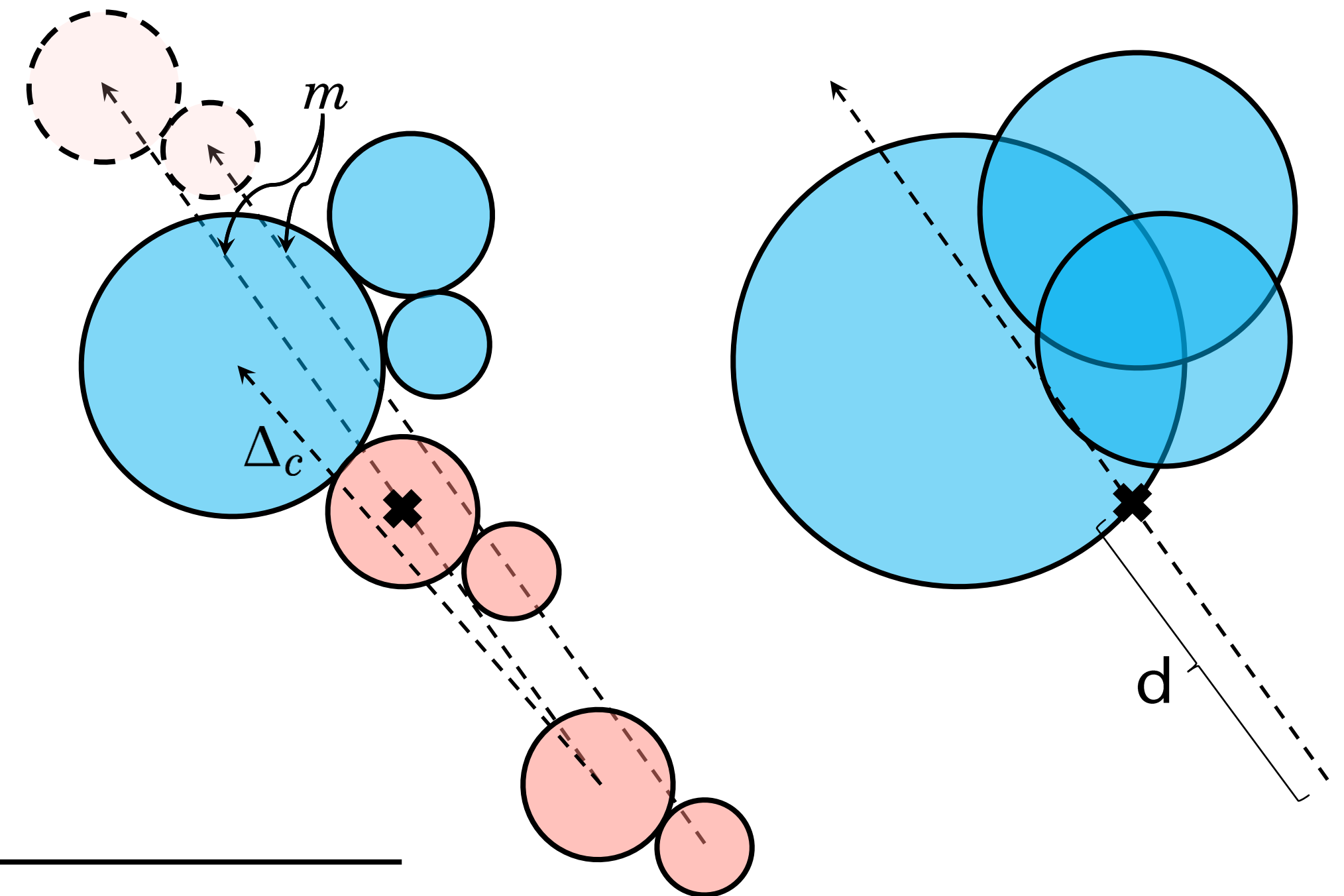


# Simulation — 1. Surface Contact

- Regard proxy  $\rightarrow$  HIP as movement
- Continuous collision detection
  - For individual spheres
  - Reduce to line-sphere intersection

$$d_{t,e} = \frac{2(\Delta_c \cdot m) - \sqrt{4(m\Delta_c)^2 - 4m^2(\Delta_c^2 - (r_t + r_e)^2)}}{2m^2}$$

- Global minimum of  $d$  = constraint movement



- Store material properties in spheres

- Material density and friction

$$\vec{n} = \frac{\sum_{T_i \cap E_j \neq \emptyset} \text{Vol}(T_i \cap E_j) \cdot n(E_j)}{\sum_{T_i \cap E_j \neq \emptyset} \text{Vol}(T_i \cap E_j)}$$

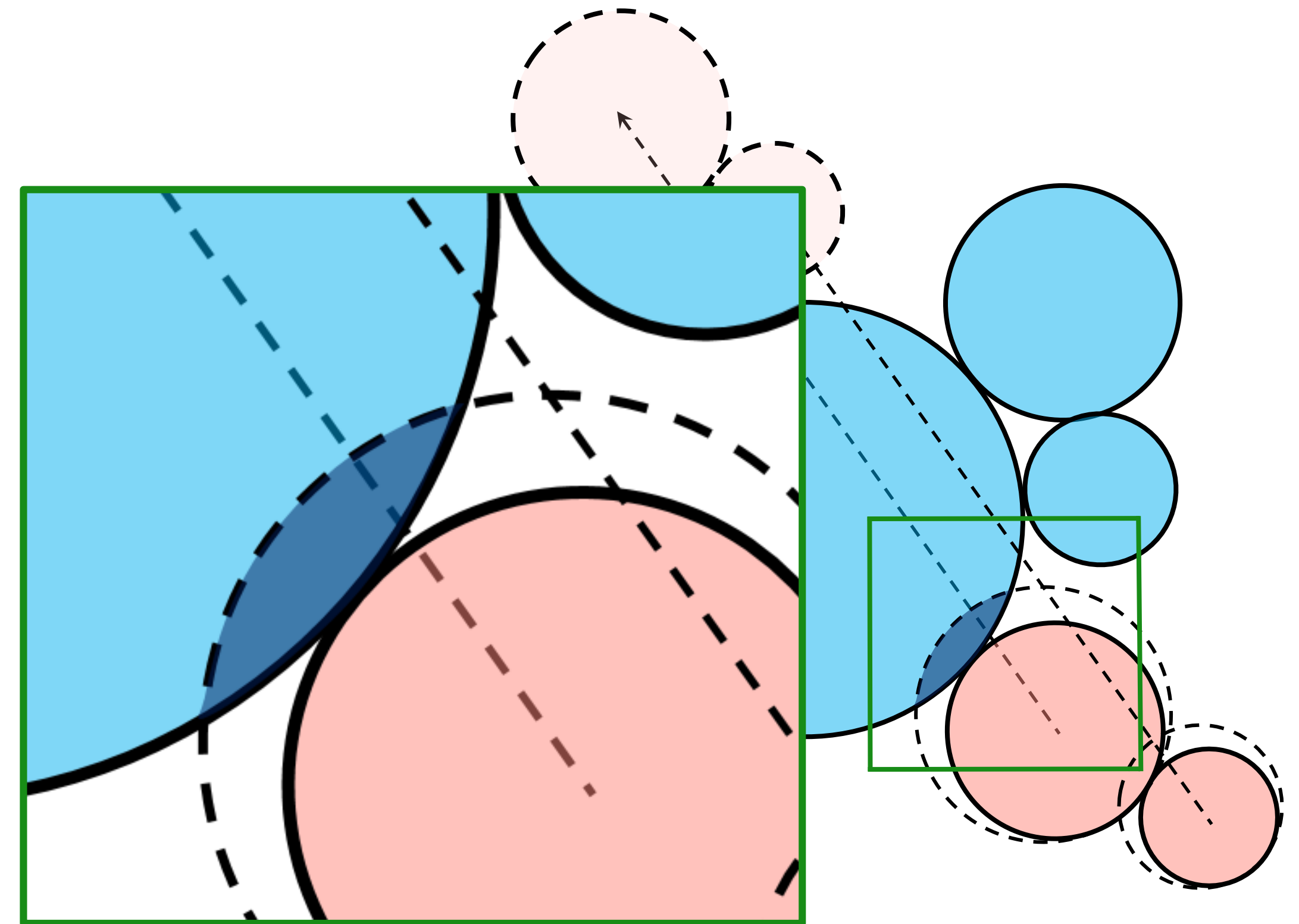
$$h = \sum_{T_i \cap E_j \neq \emptyset} \text{Vol}(T_i \cap E_j) \cdot h(E_j)$$

$$F_c := hF_c$$

- Coulomb friction

$$\vec{n}(F_c \cdot \vec{n})\mu_s < F_c - (\vec{n} \cdot F_c)$$

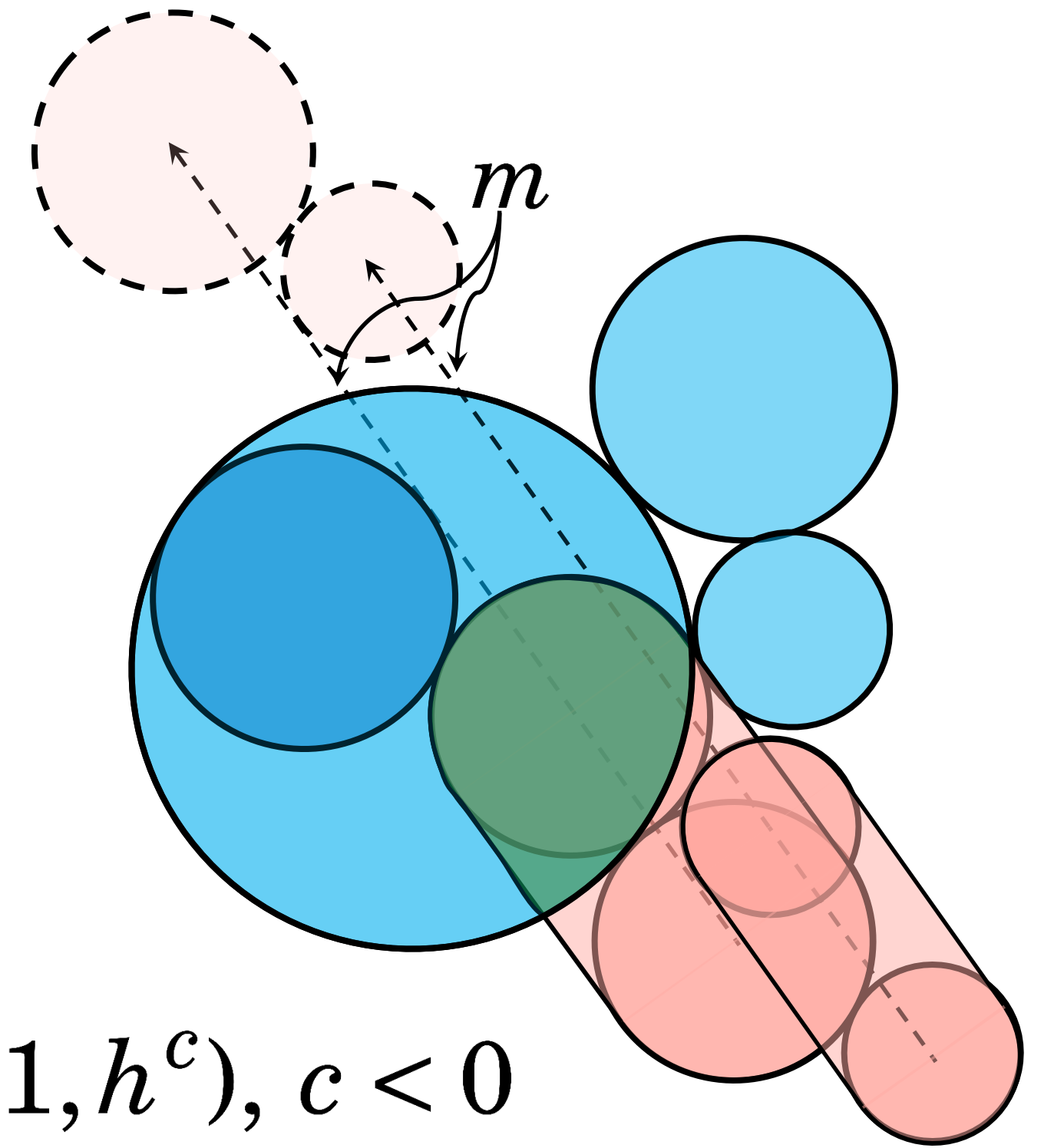
- Restrict lateral movement unless:  $\vec{n}(F_c \cdot \vec{n})\mu_k < F_c - (\vec{n} \cdot F_c)$



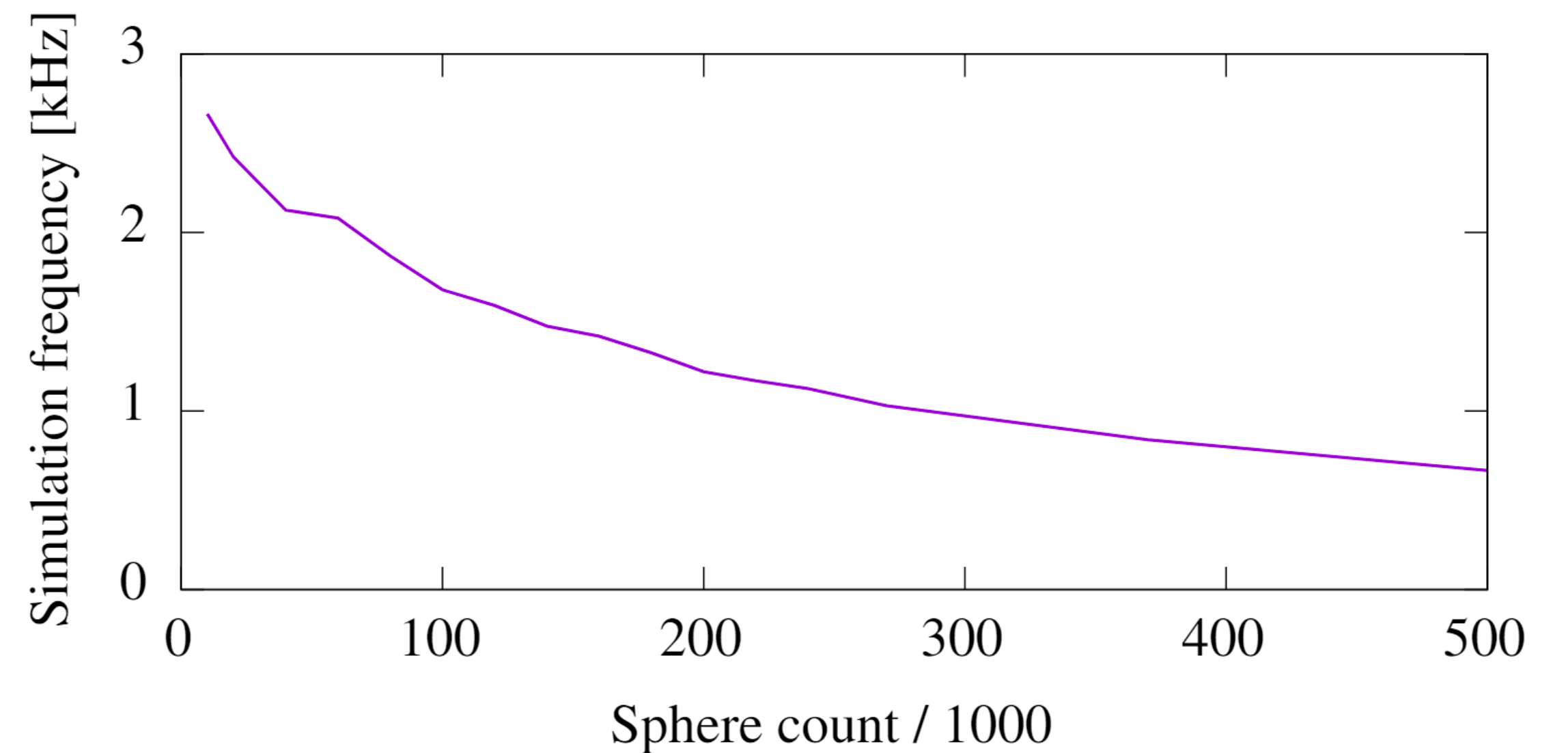
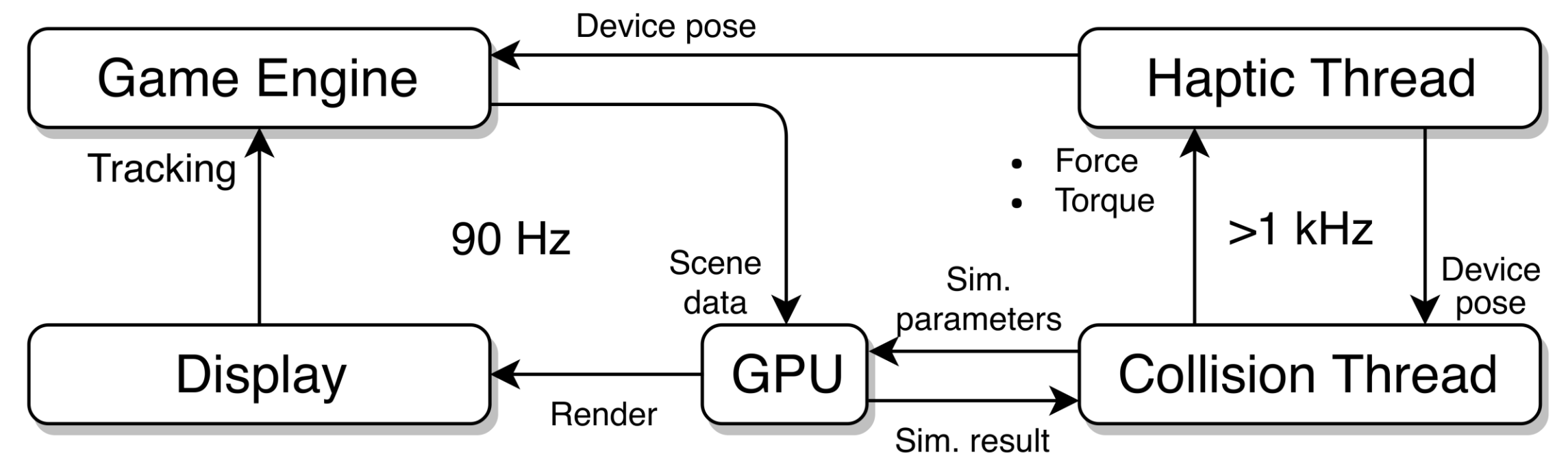


# Simulation — 3. Drilling

- Start drilling at new proxy (Pass 1)
- Estimate frame time
- Tool's drill speed  $v_d$ 
  - Scale with penetration depth updated  $m$  (Pass 1)
  - Scale inversely with density (Pass 2)  $v_d := v_d \cdot \min(1, h^c), c < 0$
- Volume updates
  - Small spheres are shrunk
  - Large spheres are subdivided



- Dynamic library
  - GPU accelerated implementation
  - Game engine plugins
- Simulation performance
  - Interaction recording
  - 1 kHz for nearly 300.000 spheres



- Simulation method with continuous drilling & stable haptic feedback
  - Fast implementation
  - Integration in popular game-engines
- Approximate 6 DoF constraints